THE

# AËRONAUTICAL JOURNAL.

(FOUNDED 1897 in succession to the ANNUAL REPORTS.)

THE ORGAN OF THE ROYAL AËRONAUTICAL SOCIETY.

*Published Monthly at the Offices of the Society, at 7, Albemarle Street, Piccadilly, London, W.1.*
*Telephone: "Gerrard 7373." Telegraphic Address: "Didaskalos, Piccy, London."*
*Subscription per annum, £1 12s. 6d.; single numbers, 2s. 9d., post free.*

**Edited for the Council by J. LAURENCE PRITCHARD, Fellow.**

*All communications should be addressed to the Editor.*

No. 143. NOVEMBER, 1922. VOL. XXVI.

## Notices of the Royal Aëronautical Society.

### Elections.

The following new members were elected at a Council meeting held on October 17th:—

*Associate Fellow.*—H. F. Parker.

*Students.*—W. H. Brown, P. B. Humphrey, C. J. Sanders and F. E. Siggers.

*Associate Member.*—M. F. Wren.

SCOTTISH BRANCH.—*Associate Member.*—Cecil Veitch.

### Examination.

The Council announce the following list of successful candidates in the Associate Fellowship Examination held on September 26th:—

*K. W. Berger (Theory of Structures, etc.).
S. H. Evans (Theory of Structures, etc., and Aerodynamics).
T. A. Kirkup (Theory of Structures, etc., and Aerodynamics).
H. J. Mackintosh (Theory of Structures, etc., and Aerodynamics).
*G. E. Page (Theory of Structures, etc.).
*S. O. Smith (Theory of Structures, etc.).
S. E. Taylor (Heat Engines and Meteorology and Navigation).

* These candidates already possessed qualifications which exempted them from the necessity of taking the paper in Aerodynamics.

### Lectures.

Owing to the continuance of rebuilding operations at the Royal Society of Arts members are asked to note that Major A. R. Low's lecture, "A Review of Airscrew and Helicopter Theory, with Aeroplane Analogies," will take place at 5.30 p.m., in the theatre of the Royal United Service Institution, Whitehall, on Thursday November 2nd.

### Donations.

Professor Leonard Bairstow has presented to the Society's Loan Collection the lantern slides used by him in his lecture on "The Work of S. P. Langley."

The Council have accepted an offer from Major Wilfred T. Blake to present to the Society's Library a copy of the notes of his flight from London to Calcutta and of a number of photographs taken en route.

**Students' Section.**

The annual meeting of the Students' Section was held in the Library, at 6.45 p.m., on October 12th, when Mr. T. C. Sharwood was appointed Honorary Secretary in succession to Mr. S. H. Evans. It was decided that each college or group of students should nominate a student to represent it and to further the interests of the Students' Section in the particular college or group concerned.

The annual meeting was followed at 7.30 p.m. by the inaugural address of the session from Dr. A. J. Sutton Pippard on "The Aeroplane Considered as an Engineering Proposition," illustrated by lantern slides.

**Forthcoming Arrangements.**

Nov. 1, *Scottish Branch*, 8.0 p.m.—Sir Sefton Brancker, Associate Fellow, "The Latest Developments of Civil Aviation," Royal Technical College, Glasgow.

,, 2, 5.0 p.m.—Major A. R. Low, Fellow, "A Review of Airscrew and Helicopter Theory, with Aeroplane Analogies," Royal United Service Institution.

,, 8, *Scottish Branch*, 8.0 p.m.—Professor Gordon Gray, "Research Work on the Application of Gyroscopes to Aeronautics," Glasgow University.

,, 9, 7.30 p.m.—*Students' Meeting.*—Mr. H. C. Brown, "Airships." Chairman, Lieut.-Col. W. Lockwood Marsh. Library.

,, 16, 5.0 p.m.—Mr. R. McKinnon Wood, Fellow, "The Co-Relation of Model and Full-Scale Work," Royal Society of Arts.

,, 21, 4.0 p.m.—Library and Publications Committee.
4.30 p.m.—Candidates' Committee.
5.0 p.m.—Council.

Dec. 6, *Scottish Branch*, 8.0 p.m.—Major J. S. Buchanan, Associate Fellow, "The Latest Developments in Aeronautical Research," Royal Technical College, Glasgow.

,, 7, 5.30 p.m.—Professor C. Frewen Jenkin, Fellow, "Fatigue in Metals," Royal Society of Arts.

W. Lockwood Marsh.

# PROCEEDINGS.

## FIRST MEETING, 59th SESSION.

The first meeting of the Session was held at the Royal United Service Institution, Whitehall, London, on Thursday, October 5th, Prof. L. Bairstow presiding.

The CHAIRMAN said that before vacating the chair and reading his paper he had a very pleasant duty to perform in the acknowledgment of work done by members of the Society during the past year. As they all knew, the Society gave medals to its members on special occasions. The gold medal had been awarded by the Wright Brothers to Mr. Busk, Professor Bryan, and one or two others, but it was very rarely given. They also had a silver medal, which, during the past eight or nine years, had been still more rarely given. The Council, however, had decided to establish a regular custom, and in future the silver medal was to be awarded annually to the person—not necessarily a member—who read before the Society the best paper, or who had the best paper published in the Journal of the Society. From possible recipients of that award had been excluded the Wilbur Wright Memorial lecturers and also those for the R.38 Memorial and the Busk and Osborne Memorials. The Council, having considered the papers presented during the past year, had decided to award the silver medal to Mr. Ricardo for his paper on "Some Possible Lines of Aero Engine Development."

The medal was presented to Mr. Ricardo amid applause.

The Chairman, continuing, said there was also a bronze medal of the Society, which was intended to recognise special merit as judged from papers published in the AERONAUTICAL JOURNAL which had been written by students. The bronze medal was not awarded this year, but it was hoped that, with the growth in the strength of the Students' Section, there would be occasion to award it in the coming year. In addition, mainly through the encouragement of one of the members of the Society, a new prize had been instituted during the year—the Pilcher Memorial Prize; this prize was also limited to students, and for this year had been awarded to Mr. Evans for his paper on "Some Notes on Commercial Aircraft."

A set of books was then presented to Mr. Evans.

The Chairman said this concluded the first part of the proceedings, and as it would be rather difficult for him both to take the chair and to read a paper, he would ask Major Wimperis to act as Chairman for the remainder of the proceedings.

Major WIMPERIS, on taking the chair, said that Col. Mervyn O'Gorman should have been in the chair, but he was not in the country at the present time as he was travelling in Italy and attending a Conference in Rome. In the absence of Col. O'Gorman, therefore, the duty fell upon himself to take the chair, and he was the more glad to do so because it gave him an opportunity to refer to the personality of the incoming President, Prof. Bairstow. He had known Prof. Bairstow a very, very long time. They were students together at about the same time in the Imperial College of Science, and he remembered that for several decades there the most brilliant student that had been produced by the College was their incoming President. He remembered that Prof. Bairstow had an uncanny faculty of making himself acquainted with, and making completely original suggestions on, subjects which they did not think he knew anything about. At any rate, people who did not know him well thought that perhaps

he did not know anything about them, but those who got to know him, as his fellow students did, were prepared for anything, and things were brought to a head, so to speak, at a meeting of the Institution of Civil Engineers, when the subject under discussion was, as far as most people knew, rather off the track of the immediate interests of Prof. Bairstow; but he showed himself not only very completely acquainted with it, but made the most fundamentally interesting and suggestive proposals for further work. Therefore the Society was to be congratulated in having at its head, during the coming Session, Prof. Bairstow. His work in the field of aeronautics was so well known that he need not dilate upon it at all, and he would therefore call upon Prof. Bairstow to give his lecture.

## THE WORK OF S. P. LANGLEY.

My special object in choosing the subject of to-night's lecture was to draw attention to a first-rate example of systematic inquiry, the type of inquiry properly called scientific. Progress was made step by step in the face of formidable difficulties, and no attempts were made to solve the problems of mechanical flight by bursts of brilliance of the type known as the invention of genius. To my mind this, the scientific method, is most suitable for the great bulk of human endeavour, and we should accept the phenomenal leaps of some individuals as the exception rather than the rule. The influence of Langley lies in the force of example and the spirit of his work rather than in the permanence of his data. It is probably not wide of the mark to say that the experimental results of Langley are now rarely appealed to, since they have been succeeded by others of greater accuracy and more immediate applicability, and yet who can doubt that the whole course of aviation was largely determined by the efforts of this one man. Without him I think it almost certain that flying would not have been ready for the Great War, with consequences which we can imagine.

Langley's work is described, largely by himself and his immediate colleagues, in two or three volumes of which the most bulky is the "Langley Memoir on Mechanical Flight," published in 1911. It is divided into two parts, the first, covering the period 1887 to 1896, written by Langley, and the second, 1897 to 1903, by Charles M. Manly, assistant in charge of experiments. The end of this period is significant, coinciding almost precisely with the earliest successes of the Wright Brothers. The record after that date has been marred by the Hammondsport trials on the modified Langley aeroplane. I want you to leave those trials out of your account, for they have nothing to do with Langley and his methods.

Writing in 1901 Langley says:—*

> "And now, it may be asked, what has been done? This has been done: A 'flying machine,' so long a type for ridicule, has really flown; it has demonstrated its practicability in the only satisfactory way—by actually flying—and by doing this again and again under conditions which leave no doubt.
>
> "There is no room here to enter on the consideration of the construction of larger machines, or to offer the reasons for believing that they may be built to remain for days in the air, or to travel at speeds higher than any with which we are familiar. Neither is there room to enter on a consideration of their commercial value, or of those applications which will probably first come in the arts of war rather than those of peace; but we may at least see that these may be such as to change the whole conditions of warfare, when each of two opposing hosts will have its every movement known to the

* "The Langley Aerodrome."

other, when no lines of fortification will keep out the foe, and when the difficulties of defending a country against an attacking enemy in the air will be such that we may hope that this will hasten rather than retard the coming of the day when war shall cease."

This note was written before the advent of the man-carrying aeroplane—two years before. Some of the prediction is yet unfulfilled, particularly that as to remaining for days in the air, but the effort of imagination which expected war developments before civil, and the difficulties of defence against hostile aircraft, are now matters of general knowledge and common anxiety. He continues his story in a different but not less interesting strain.

"I have thus far had only a purely scientific interest in the results of these labours. Perhaps if it could have been foreseen at the outset how much labour there was to be, how much of life would be given to it, and how much care, I might have hesitated to enter upon it at all. And now reward must be looked for, if reward there be, in the knowledge that I have done the best in a difficult task, with results which it may be hoped will be useful to others. I have brought to a close the portion of the work which seemed to be specially mine—the demonstration of the practicability of mechanical flight—and for the next stage, which is the commercial and practical development of the idea, it is probable that the world may look to others. The world, indeed, will be supine if it do not realise that a new possibility has come to it, and that the great universal highway overhead is now soon to be opened."

In that short passage is much of interest; it points out the unknown amount of work involved in a particular piece of research, and that the reward is often in internal satisfaction and not commercial return. To the scientist the financial returns are, I think, rightly, less appreciated than the successful campaign against difficulties, though he must, since he belongs to the animal kingdom, pay such attention to the former as will keep body and mind fit for the task. Bodily, that is physical, fitness is as great an asset in scientific inquiry as in other branches of life, and few of us fail to realise some of the more obvious correlations of mind and matter dealt with scientifically by the President of the Royal Society at the meeting of the British Association for the Advancement of Science held at Hull this year.

I think Langley might well be satisfied with the help that he has given to others, though he might wish to reiterate his concluding paragraph to a world which has allowed 20 years to lapse without realising the value of the opening of the great universal highway. Possibly, however, he might exhibit some of the patience shown in his experimental work and see sufficiently steady if slow progress towards the goal he foresaw. On the commercial aspect, Manly has a word to say in his preface to the "Langley Memoir."

"Persons who care only for the accomplished fact may be inclined to under-rate the interest and value of this record (1911). But even they may be reminded that but for such patient and unremitting devotion as is here enregistered, the new accomplished fact of mechanical flight would still remain the wild unrealised dream which it was for so many centuries.

"To such men as Mr. Langley an unsuccessful experiment is not a failure, but a means of instruction, a necessary and often an invaluable stepping-stone to the desired end. The trials of the large aerodrome in the autumn of 1903, to which the curiosity of the public and the sensationalism of the newspapers gave a character of finality never desired by Mr. Langley, were to him merely members of a long series of experiments, as much so as any trial of one of the small aerodromes or even one of the earliest rubber-driven models. Had his health and strength been spared, he would have

gone on with his experiments undiscouraged by these accidents in launching and undeterred by criticism and misunderstanding.

"Moreover, it is to be borne in mind that Mr. Langley's contribution to the solution of the problem is not to be measured solely by what he himself accomplished, important as that is. He began his investigations at a time when not only the general public but even the most progressive men of science thought of mechanical flight only as a subject for ridicule."

It appears that the pursuit of knowledge requires some courage, and Langley impresses one as having been able to recognise home truths with detachment and humour. In one passage he says:—"It has taken me, indeed, but a few years to pass through the period when the observer hears that his alleged observation was a mistake; and the period when he is told that if it were true it would be useless; and the period when he is told that it is undoubtedly true, but that it has always been known." It sounds like modern history instead of 25 years ago, and I wonder how many cold shivers are passing down our backs at the thought that we may be saying such things ourselves. Of course it is not likely, but then———. Even Langley had periods of doubt and a feeling that he was beating the air, and he singles out the year 1895 as one which "gave small results for the labour with which it was filled"; and then comes the buoyancy:—"Shortly after its close, I became convinced that substantial rigidity had been secured for the wings; that the frame had been made stronger without prohibitive weight, and that a degree of accuracy in the balance had been obtained which had not been hoped for. Still there had been such a long succession of disasters and accidents in the launching that hope was low when success finally came."

Just one more quotation before becoming immersed in technics. The general unbelief of the world's inhabitants had its effect on Langley's experiments. He shrank into his shell, and Mr. Alexander Graham Bell records the "pleasure of witnessing the successful flight of some of these aerodromes more than a year ago, but Dr. Langley's reluctance to make the results public at that time prevented me from asking him, as I have done since, to let me give an account of what I saw." Langley himself makes it clear that in his view the public are not good observers of experiments, and says, "It is the practice of all scientific men, indeed of all prudent men, not to make public the results of their work till these are certain." That he was ready to give to the world complete accounts of his experiments, both successes and failures, without commercial motive, is clear. It may be that some such feelings on the part of scientific men lead to their withdrawal from public life, action which many of us deplore but understand. I don't suppose that it is the only reason, and with an awakening of business life to the value of science perhaps we may hope for an atmosphere in which both scientists and men of commerce may meet and continue to meet.

Langley dates his interest in flight as beginning in the years 1886-1887, whilst he was engaged in the study of astrophysics in Allegheny, Pa. At that time most of those now connected with aviation were children at school picking up some of the more elementary knowledge transmitted to us from past generations. Not only was it 35 years ago in total time, but it was rather more than 20 years prior to the first public exhibitions of man-carrying aeroplanes. Langley himself died in 1906, and so just failed to see a definite seal put on his speculations as to the future, although he was aware of the secret flights of the Wright Brothers, and took a great interest in their work.

The position at that time is clearly summarised by Langley:—

"I desire to ask the reader's consideration of the fact that even 10 years ago (*i.e.*, prior to 1897) the whole subject of mechanical flight was so far from having attracted the attention of physicists or engineers, that it was generally considered to be fitted rather for the pursuits of the charlatan

than for those of the man of science. Consequently, he who was bold enough to enter it, found almost none of those experimental data which are ready to hand in every recognised and reputable field of scientific labour. Let me reiterate the statement, which even now seems strange, that such disrepute attached so lately to the attempt to make a "flying machine" that hardly any scientific men of position had made preliminary investigations, and that almost every experiment to be made was made for the first time. To cover so vast a field as that which aerodromics is now seen to open, no lifetime would have sufficed. The preliminary experiments on the primary question of equilibrium and the intimately associated problems of the resistance of the sustaining surfaces, the power of the engines, the method of their application, the framing of the hull structure which held these, the construction of the propellers, the putting of the whole in initial motion, were all to be made, and could not be conducted with the exactness which would render them final models of accuracy."

Throughout his writings Langley made a clear distinction between two subjects which he called "aerodynamics" and "aerodromics," a distinction which still exists, but is differently described. His divisions correspond very closely with the modern expressions "performance" and "control and stability," both of which are now regarded as branches of aerodynamics. The scientific advisers of the Air Ministry are more and more turning to the study of "aerodromics" on which progress towards safety is seen to depend very largely. Its problems are very difficult at the present time, and in the absence of scientific executive direction progress will continue to be slow.

There is little present evidence of the spirit of Langley, which takes up a task so great that "to cover it no lifetime would have sufficed." Team work may be continuous, but the work of an individual is necessarily limited by the length of his active years of life. If, however, the team is to be steady it must be well guided, and we have yet much to learn in the methods of doing this. Would Langley, who saw so clearly the broad future of aviation, or a non-scientific man have been the better director of research?

Having so divided the field of inquiry, Langley first devoted his whole attention to aerodynamics and to the establishment of certain fundamental data on which to base estimates of the performance of heavier-than-air craft. His chief piece of apparatus was a whirling table 60ft. in diameter, near the Allegheny Observatory; in 1889 it was surrounded by an octagonal fence 20ft. high open to the heavens in an attempt, which proved to be ineffectual, to ward off some of the disturbances due to wind. His last word of advice on this subject was: "If any one should propose to repeat or extend these experiments, I would advise him, first of all, and at all costs, to establish his whirling table in a large, completely inclosed building."

When, in 1909, the Advisory Committee for Aeronautics was formed, and decided to build a whirling table at the National Physical Laboratory, this advice was followed with success. Time has, however, shown that the wind channel is a much more suitable piece of apparatus for the purpose, and we now have that fund of knowledge, the lack of which was felt so acutely by the early pioneers. It is to this fact in large measure that we must attribute the possibility of appreciable numbers of designers and workers in aviation and not to any development of unusual character in the powers of the human mind.

After trying to work the whirling arm with a gas engine of 1½ h.p. without satisfaction, a steam engine of 10 h.p. was used from October, 1888 onwards, and with it speeds of from 10 to 70 m.p.h. were attained. The resources of the observatory were called upon, particularly with regard to chronographs, but the main pieces of apparatus were specially designed for the inquiry afoot. The possible errors due to circular motion of his test sections instead of a rectilinear one were dis-

cussed, and realised to be of a nature insusceptible of exact definition. Whilst ready to theorise wherever possible, Langley also clearly recognised the limitations of theoretical argument, an attitude which is still one of importance. Those of you who have taken an interest in the press accounts of Prof. Sherrington's address to the British Association will have seen how far lack of this power of discrimination can carry individuals. To read into the address on the relation between mind and certain parts of the brain, any idea that the working of the mind has been reduced to an understandable process would, I think, not occur to a scientist.

In carrying out his work, Langley was evidently troubled by the existence of a theory by Newton on the aerodynamic forces on an inclined surface. One can understand a state in which his experiments were discounted by others because they were at variance with Newton's sine-squared formula and the reaction indicated in the passage, "It is important to remember that the mathematical method as applied to physics, must always be trustworthy or untrustworthy, according to the trustworthiness of the data which are employed; that the most complete presentation of symbols and processes will only serve to enlarge the consequence of error hidden in the original premises, if such there be, and that here, as will be shown, the error as to fact begins with the great name of Newton himself."

Many defences of Newton have been attempted which reduce to the argument that his reasoning was faultless; that of course wholly fails to meet Langley's criticism; a little later we shall find an error of the same type in Langley's own deductions, and I for one prefer to accept them as they stand, examples of the limitations of the best of human beings. It is a comfort at times to be able to turn to the works of our great men and to find evidences of a frailty more marked in ourselves, but yet of the same character. It is an antidote to the oft-repeated saying that the last generation is better than the present. I suppose the saying was true at certain periods, the decline and fall of the Roman Empire, etc., and it may come to be really true here, but in all probability the remark generally only means a forgetting of the errors of the past, and a remembering of the better qualities. It is generous to the past, hardly just to the present.

The experiments devised by Langley are of a type not now used, and the deductions from them are rarely so direct as those made in an aerodynamics laboratory. This is partly due to the need for such experiments as could be made self-recording, direct observation at the end of a whirling arm not being feasible. Some first approximations to the measurement of lift and drag were made by the "suspended plane." This was a plate free to blow back under the influence of the wind about an axis through the upper edge. It was fairly heavy, two pounds, and so the angle taken up depended on the speed of the wind. The plate was suspended from its upper edge by a spring, the extension of which measured the force along the guides of the frame, *i.e.*, during an experiment gave the difference between the weight of the plane and the lift on it. Automatic records of the extension of this spring and the angle taken up by the plate at various speeds constituted the fundamental observations. In the reduction of them Langley appears to have assumed that, other things being the same, the aerodynamic reaction varied as the square of the speed, and that the pressure of a fluid is always normal to a surface moving in it. Neither of these is strictly true, the first ignoring the scale effect, first accurately indicated by the theorem of dynamical similarity, and put into form by the Lord Rayleigh; and the second being true only for a frictionless fluid. For the greater number of experiments made by Langley, the errors in the two assumptions would be small, but the second led to an important error at high speeds. Langley seems to have satisfied himself that the tangential component of the resultant force was negligible under all practical circumstances, and amongst the first to discover the error in this was Mr. F. W. Lanchester, who questioned the accuracy of the

statement by Langley that "a definite amount of power . . . will attain more economical results at high speeds than at low ones . . . up to some remote limit not yet attained in experiment but probably represented by higher speeds than have as yet been reached in any other mode of transport."

There is a caution attached to this remark as to a limit, but there is little doubt that Langley overestimated the advantages of high speed, and that the error was an error of deduction rather than observation. I have reproduced from Fig. 11 of his book on "Experiments in Aerodynamics," a diagram to illustrate this point. The ordinate of the curve is the resistance of an inclined plane of aspect ratio rather greater than 6, whilst the abscissæ are angles of incidence ranging up to 45 degrees. The plate weighed 500 grammes, and was moved round at "soaring speed" for each angle of incidence, *i.e.*, the lift on the plane was constant and equal to 500 grammes.

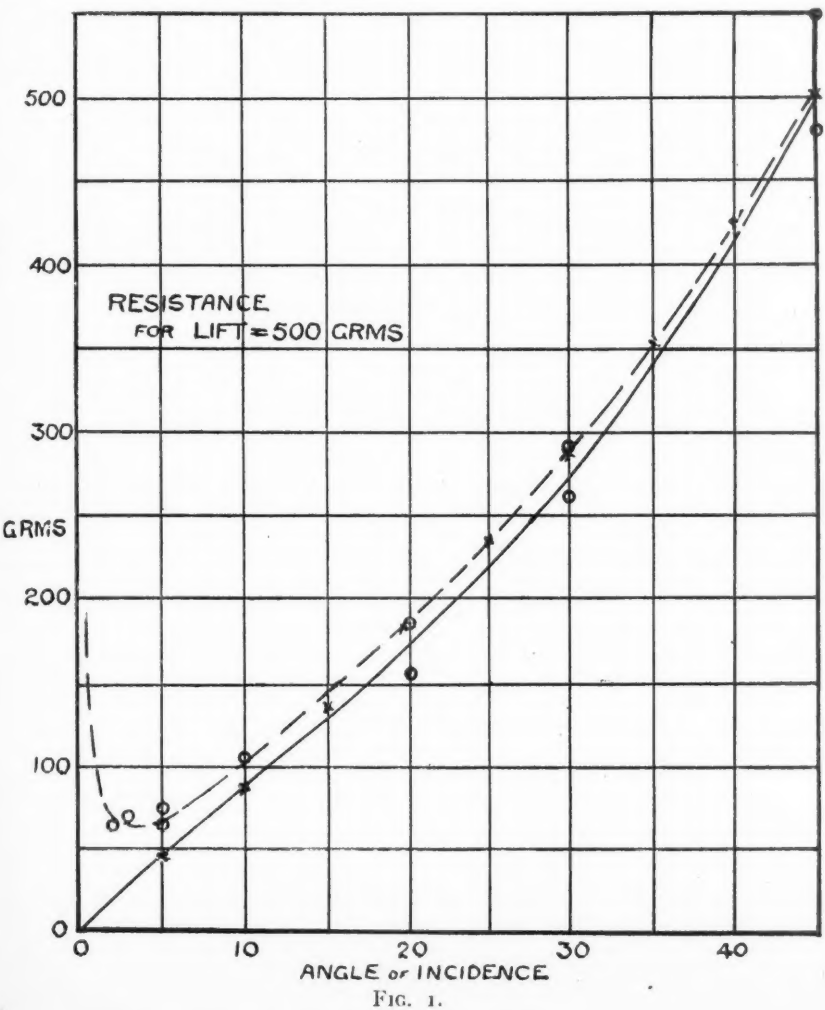



FIG. 1.

Observations are indicated by small circles, and a relatively high degree of accuracy is shown by the general consistency. Crosses were obtained by calculation on the hypothesis that the resultant force on the plane is normal to the surface, and it will be seen that at large angles of incidence the observed points lie close to the curve theoretically anticipated at that time. The proportionate discrepancy becomes important at angles of 5 to 10 degrees, and here Langley's trust in theory appears to have been great enough to mislead him. He says " For angles below 10 degrees, the curve (full line of figure), however, instead of following the measured pressure, is directed to the origin, so that the results will show a zero horizontal pressure for a zero angle of incidence. This, of course, must be the case for a plane of no thickness, and cannot be true for any planes of finite thickness with square edges, though it may be and is sensibly so with those whose edges are rounded to a so-called ' fair ' form." In coming to this conclusion, Langley had made calculations of the normal force on the edge and the skin friction according to the then known formula, but as we now know, the conclusion was erroneous.

It led him to estimate the high value of 25 for the ratio of lift to drag of a plane at an angle of incidence of 2 degrees instead of a more probable 7 or 8. At angles greater than 10 degrees the error was not serious.

From the date given in his papers I have extracted sufficient to exhibit the results in diagrams of modern form. From Fig. 2 it will be seen that the curve of lift coefficient as a function of angle of incidence is of the normal form and that the maximum value of $k_L$ of 0.4 agrees with more recent observations and indicates the general soundness of Langley's preliminary work. The curve for lift/drag is also of normal character, and in Fig. 2 observations are shown by circles. Langley's estimate corresponds closely with the theoretical assumption of zero skin friction, which leads to the highest curve of Fig. 2. At an angle of incidence of 5 degrees the estimate for lift/drag was not so high as 12, and we now know that values greater than this are utilisable with cambered planes of type fitted by Langley to his models.

Unfortunately, Langley stressed this point to one of extreme prominence in his work, and it has been commented on by critics ever since, rather to the exclusion of a fairer summary of the whole work. We find the following deduction from the above results :—" In this connection I may state the fact, surely of extreme interest in its bearing on the possibility of mechanical flight, that while an engine developing one horse-power can, as has been shown, transport over 200 pounds at a rate of 45 m.p.h., such an engine (*i.e.*, engine and boiler) can be actually built to weigh less than one-tenth of this amount." This deduction is based on flight at 2 degrees and a lift to drag ratio of 25. In spite of his optimism, it may be seen that results which encouraged Langley to proceed would have reduced present-day designers to despair.

In his section on aerodynamics, other experiments of interest were made, some on the centre of pressure of a square plane, and also on its resistance in normal presentation.

In summarising his conclusions, Langley has matters of interest for us. For example, he says :—

" I am not prepared to say that the relations of power, area, weight, and speed, here experimentally established for planes of small area, will hold for indefinitely large ones; but from all the circumstances of experiment, I can entertain no doubt that they do so hold far enough to afford assurance that we can transport (with fuel for a considerable journey and at speeds high enough to make us independent of ordinary winds) weights many times greater than that of a man."

And again :—" I desire to add as a final caution, that I have not asserted that planes such as are here employed in experiment, or even that planes of

any kind, are the best forms to use in mechanical flight, and that I have also not asserted, without qualification, that mechanical flight is practically possible, since this involves questions as to the method of constructing the mechanism, of securing its safe ascent and descent, and also of securing the indispensable condition for the economic use of the power I have shown to be at our disposal—the condition, I mean, of our ability to guide it in the desired horizontal direction during transport—questions which, in my opinion, are only to be answered by further experiment, and which belong to the inchoate art or science of aerodromics, on which I do not enter."

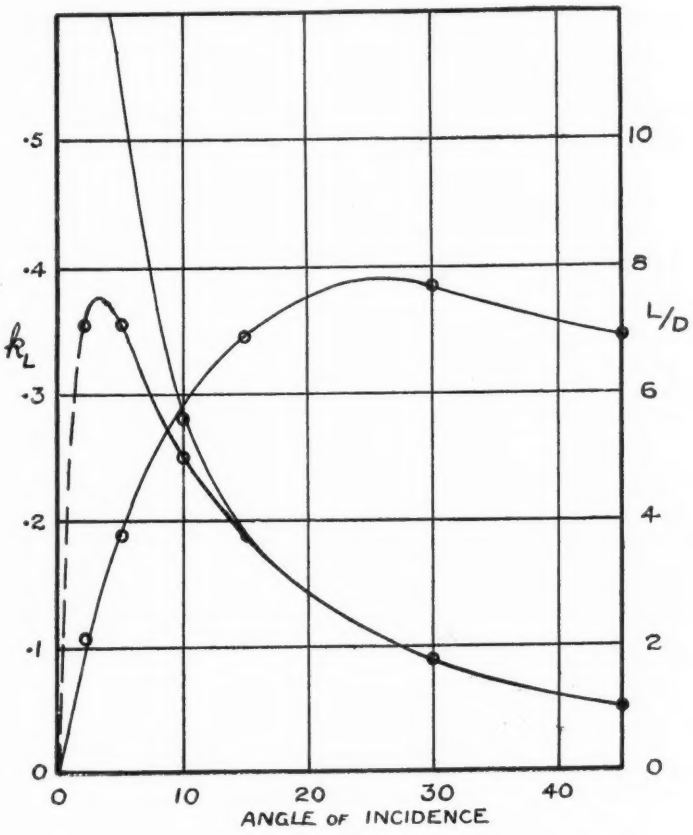



FIG. 2.

These remarks were made in 1891. They show a substantial, but not unblemished, record which was to serve as a basis for further work. Langley did indeed leave the subject of aerodynamics as he defined it, *i.e.*, performance in a secondary place whilst he attended to aerodromics or control and stability. He aimed at demonstrating the possibility of mechanical flight to a general public, and to him there was no noteworthy distinction between demonstration on a model or on a man-carrying aeroplane. In either case he thought that the aircraft ought to be capable of keeping itself in correct attitude in the air, and set himself to find the appropriate arrangements.

It is a long story, from which I propose to make extracts in due course, but in the end the demonstration was made. The model had two pairs of wings in tandem with a span of between 12 and 13ft., and an overall length of about 16ft. It weighed nearly 30lbs., of which about one-fourth was in the engine. The steam engine developed 1 to 1½ h.p., and weighed only 26 ounces; it was fed by a boiler, which with its firegrate weighed a little over 5lbs. The wing area was 54 sq. ft., and the section had a camber of one in twelve.

As compared with the modern aeroplane the model was extremely lightly loaded (.55lbs. per sq. ft.) as against the more common present value of 7 or 8. The weight carried per h.p. was from 20 to 30lbs., a value very similar to that of bombing aircraft, or the fully-loaded civilian aeroplane. Langley's figures had not led him far wrong, for a reason that appears in one of the estimates of performance. Elements are reproduced below:—

| | |
|---|---|
| Flying weight ... ... ... ... | 22lbs. |
| Area of wings ... ... ... ... | 40 sq. ft. |
| Engine power ... ... ... ... | 1 h.p. |
| Power necessary to fly ... ... ... | 0.35 h.p. |
| Flying speed for wings at 20 degrees ... | 16-17 m.p.h. |

As we saw a little time ago, 20 degrees is well inside the experimental range of the aerodynamic experiments and not liable to the error pointed to earlier. Further, Langley provided three times as much power as he estimated was necessary for flight, to cover inefficiency of gearing, airscrews, and errors in estimation. A number of minor flights were recorded, but the seal seems to have been set on May 6th and November 28th, 1896. Of them Mr. Manly says: "For the first* time in the history of the world a device produced by man had actually flown through the air, and had preserved its equilibrium without the aid of a guiding human intelligence. Not only had this device flown, but it had been given a second trial and had again flown, and had demonstrated that the result obtained in the first test was no mere accident."

On May 6th, 1896, Aerodrome No. 5 was launched directly into the gentle breeze which was then blowing. "The height of the launching track above the water was about 20ft. Immediately after leaving the launching track, the aerodrome slowly descended three or four feet, but immediately began to rise . . . to circle to the right, and moved around with great steadiness, traversing a spiral path as shown in the diagram.† During the first two turns the machine was constantly and steadily ascending, and at the end of the second turn it had reached a height variously estimated . . . at from 70 to 100ft. When at this height, and after a lapse of one minute and twenty seconds, the propellers were seen to be moving perceptibly slower, and the machine began to descend slowly. . . ." The distance traversed was a little over 3,000ft. at a rate of from 20 to 25 m.p.h. A second flight was made on the same day, which was also successful and a well earned reward of indomitable perseverance.

The record for November 28th, 1896, refers to Aerodrome No. 6, and the path is shown in the diagram. The flight extended for about three-quarters of a mile at a speed of approximately 30 m.p.h., and the success achieved led to the attempt of the next step, the making of a man-carrying aeroplane; the pilot was to be Mr. Manly. This stage had only been reached by much effort and care, and in the process ten years had gone. It was then that interest in

* This quotation is not in agreement with the records of aviation as found in the library of the Royal Aeronautical Society. In the chronology of "A Brief Account of the Aeronautical Society" will be found the reference—1848. Stringfellow succeeded in making his large model aeroplane to fly with a small steam engine. Further short flights appear in Hargraves' records of date 1884 or 1885.

† A number of slides were used to illustrate the lecture which are not reproduced here. In all cases the originals of the slides appeared in the Langley Memoir.

flying widened, slowly at first, but growing in strength until, less than twenty years after these model flights, the whole world was absorbed in the subject, and the records of airmen at the front thrilled every nation. We have gone back a little, let us hope to gather strength for the next forward move, but few now exist who envisage a return to the time when no aircraft existed. How much of this is due to Langley and his successful models? I don't suppose we can assess the relative value, but some of us at least feel that he bore the brunt of the fray for a long period without falling by the wayside, and so left us a legacy of example to be followed.

Before closing with a description of the man-carrying aeroplane—a description of qualified success, almost failure—a few words may be desirable on the early development. A number of slides, prepared from the figures in the "Langley Memoir," will perhaps help in making points clear.

"When the details of the aerodrome . . . are considered from the standpoint of the engineer accustomed to make every provision against breakage and accident, and to allow an ample factor of safety in every part, they will be found far too weak to stand the stresses that were put upon them. . . . It was absolutely necessary, in order to insure success, that the weight should be cut down to the lowest possible point, and when this was reached it was found that the factor of safety had been almost entirely done away with, and that the stresses applied and the strength were almost equal."

Of course this applied to steady flying, but now, after the expenditure of many millions of pounds in the development of military aircraft, the factor of safety against stunting is not so great as two.

"In the engine the three points aimed at in the design were lightness, strength and power, but lightness above all, and necessarily in a degree which long seemed incompatible with strength."

The steam was generated in a copper coil by the impinging on it of a flame from a burner working on the principle of the familiar blow lamp used by the plumber. The chamber *D* contained compressed air for forcing the gasoline from the tank *I* into the burner *N*. The flame played on the boiler coils *O* and the stack *Q* carried away the burnt gases. The steam passed from the boiler through a separator *M* before going to the engines. The latter are seen to be across the model (Pl. 29a), and were directly connected to the airscrew shafts. A synchronising shaft and bevel gears were fitted at the front of the frame.

In a note it is said "very exact accuracy in these minute details is indispensable to the efficient working of the engine." The power weights work out as follows :—

| | |
|---|---|
| Engine ... ... ... ... ... | 464 grammes |
| Pump and pump shafts ... ... | 231 ,, |
| Gasoline tank and valves ... ... | 178 ,, |
| Burners ... ... ... ... ... | 360 ,, |
| Boilers, frames and mica covers ... | 651 ,, |
| Separator, steam gauge and pipe ... | 540 ,, |
| Exhaust pipe ... ... ... ... | 143 ,, |
| Smoke stack ... ... ... ... | 342 ,, |
| | 2,909 grammes or 6.4lbs. |

In addition, the fuel at the beginning of the flight weighed 250 grammes and the water 2,350 grammes.

It is impossible for any one closely connected with the difficulties of experiment not to admire the skill with which so much detail was handled and the persistence which led to the passing of rock after rock in a new passage. Wrecks there were, but not a succession in the same place.

The photographic records of the flight are not so good as those of August 8th, 1903, on a quarter size model of the ill-fated man-carrying aeroplane.

On the subject of balancing the aerodrome, the memoir is interesting, and regards the problems in a different light from the present. "Equilibrium may be considered with reference to lateral or longitudinal stability. The lateral part is approximately secured with comparative ease, by imitating Nature's plans, and setting the wings at a dihedral angle, which I have usually made 150 degrees (15 degrees in present use of the term)." Langley then says: "I pass on to the far more difficult subject of longitudinal stability." After discussing the movement of centre of pressure and the effect of the position of the centre of gravity he concludes that "These rules are purely empirical and only approximate. As approximations, they are useful in giving a preliminary balance, but the exact position is rarely determinable . . . except by experiment in actual flight."

I wonder how many of us, with all the data of the moment at our disposal, would be prepared to guarantee the first flight of a new aeroplane launched without pilot. Very few, I think, and those only optimists. In this respect we have a lot to learn, did the powers-that-be recognise the fact.

**The Man-Carrying Aeroplane.**

I propose to be very brief and to make my points mainly by reference to illustrations, which show the breadth of view of Langley and his colleagues. The experiments were made in a period when "his physician had counselled him that a resumption of concentrated thought and vigorous endeavour would materially shorten his life, which had already passed three score years." I wish that his further endeavour had received the crown of success given to his former efforts; it would have been a reward to him of the kind which he would have appreciated, but there will always be uncompleted tasks in a man's life history. He did a great deal and the little more would probably have made little difference to aviation, for the fires had been lighted and were beginning to burn freely. Only as a public acknowledgment do I wish that Manly and the Langley aerodrome had managed to fly for a mile or so during the trials of 1903. Instead, we find Manly attributing the end of Langley's experiments to the lack of immediate success and a consequential absence of financial support.

The outlines of the man-carrying glider are thus given:—"Starting with the assumption that Models Nos. 5 and 6 were capable of transporting a load of approximately 10 lbs. more than their weight, it was seen that, since the supporting surface of an aerodrome would increase approximately as the square of the linear dimensions, in order to carry a man the aerodrome would need to be approximately four times the linear dimensions of these models. Calculations based on the results accomplished in the construction of the models indicated that such an aerodrome would need to be equipped with engines developing 24 h.p. The best that could reasonably be hoped for was that these engines would not weigh over 200 lbs., and therefore, allowing 40 lbs. for fuel and fuel tanks, it became necessary to bring the weight of frame, supporting surfaces, tail, rudder, propellers, and every other accessory within 250 lbs. if the total weight of the machine, including 150 lbs. for the aeronaut, was not to exceed 640 lbs., or 16 times the combined weight of the model and its load of 10 lbs. Although the problem of constructing the frame, wings and all other parts within the limit of 250 lbs. seemed indeed formidable, it was believed that the greatest obstacle in the production of such a machine would be that of securing a sufficiently light and powerful engine to propel it."

In other respects, as in the balancing of the aeroplane, did Langley appreciate the difficulties, and his first intention was to fly with a dummy pilot in order to avoid possibly fatal consequences. The construction appears to have been begun in the summer of 1898, and the two unsuccessful attempts at flight occurred rather more than five years later in the autumn of 1903.

In order to give the large aeroplane the best opportunity, experiments were made on the flying of a model. "Before making the tests of the large aerodrome, it was intended to give the quarter-size model a preliminary trial to test the balancing which it was proposed to use on the large machine. . . for it was assumed that if the quarter-size model, which was an exact counterpart of the large machine, should fly successfully with the same balancing as that calculated for the larger one, the large one could reasonably be expected to act similarly." Photographs of the flights of this model were obtained on August 8th, 1903. In addition to these tests for balance, great care was exercised in the engineering design, Manly having been chosen expressly for his qualifications in this direction. The plates of details of the structure will show a result comparable with modern efforts. The results of experience preclude extension of this statement to the overall design, for in certain respects the type is far less effective for its purpose than the present biplane and small tail.

Two trials were attempted, and photographs are shown which indicate failures immediately after launching. In the earlier trial of October 7th, 1903, failure of the launching gear is definitely asserted; but in the second on December 8th, 1903, the early failure is very complete. Mr. Manly says, "The all-important question as to just what caused the accident which did occur remains to some extent a mystery." The structure is surmised to have failed at the end of the launching run by the loss of the tail and the crumpling of the rear wings. The aeroplane rose to the vertical and fell over backwards into the water.

This was the end of the series of experiments, for the Hammondsport trials were not part of the work of Langley, and in the opinion of many of us were ill-advised. The end did not come from lack of spirit, for "Doctor Langley considered it desirable to continue the experiments, but the Board deemed it advisable, largely in view of the adverse opinions expressed in Congress and elsewhere, to suspend operations in this direction." And again, "In the spring of 1904, after the repairs to the main frame were well under way, the writer (Mr. Manly), on his own initiative, undertook to see what could be done towards securing for Mr. Langley's disposal the small financial assistance necessary to continue the work; but he found that while a number of men of means were willing to assist in the development of the aerodrome, provided arrangements were made for later commercialisation, yet none were ready to render the assistance from a desire to assist in the prosecution of scientific work." On the other hand, Langley "had given his time and his best labours to the world without remuneration, and he could not bring himself at his stage of life to consent to capitalise his scientific work."

Is not this sketch rather an indication that Boards—including Air Ministries—cannot utilise the enthusiasm of scientists under their control? What is wrong? Is it the Air Ministry or the scientist, or both? Can an administrator who has no scientific knowledge direct the work of scientists? Has Britain ever allowed a fair trial to the executive control of scientific work by a man of science? How often have men of science been placed on advisory bodies and their advice ignored? I am not going to answer my own questions; in one aspect the ground is political and concerns our system of government, but in another it is a proper subject for the concern of a scientific and technical body like the Royal Aeronautical Society.

I have made my lecture rather long without covering the many interesting points in Langley's work in more than a sketchy manner. His speculations on soaring are worth the attention of any serious investigator who is trying to account for the phenomenal success of gliders.* I may not stay to deal with the subject now, for the digression, to be worth while, would be long.

* At the moment of revision of the proofs of this Paper it is interesting to note that the world's record for duration has just been obtained by M. Maneyrol on a glider having the tandem monoplane arrangement of wings characteristic of the Langley man-carrying aeroplane.

In conclusion, it appears to me that Langley belongs to the small band of pioneers who dared to attempt the seemingly impossible. Quite recently we have had a further attempt to climb Everest, an effort which is also dubbed failure. The climbing of 27,000 feet is not a matter for enthusiasm; the extra 2,000 feet would have been counted success, and yet I doubt the equity of the distinction. Surely the attempts which led to 27,000 feet and the examination of the limitations of human endurance are great contributions to the effort of that group of men who will one day stand on the summit of Everest! Surely Langley's place is of the same character as that of the climbers of Everest! He made it much easier for those to come; he helped the Wrights, the Bleriots and the Farmans of the succeeding decade to reach the summit and to increase the time for which they could maintain themselves there. Such men as Langley are not to be expected frequently in the history of a nation; his example is not an easy one to follow. I have taken a great pleasure in preparing this account of his labours and in attempting to interest others in an understanding of the processes by which aviation has come. The lecture has been exclusively devoted to Langley without any desire to make or institute comparison with other pioneers. This is not for lack of appreciation but is due to limitations of time and space.

## DISCUSSION.

Major WIMPERIS said he was sure they had all listened to Prof. Bairstow's lecture with very great interest. He had given them a record of indomitable perseverance, and that was a very welcome tonic. Moreover, the lecturer had not hesitated to lay bare certain—perhaps minor—points in which Langley really did go astray. It was a coincidence that their new President was also lecturer that evening, but he thought it was a happy coincidence, and the Council had decided to adopt that plan in the future—that the inaugural meeting of each new Session should be occupied by a lecture or paper by the incoming President. At some societies where that custom prevailed it was also the custom that the incoming President's words must be subjected to no form of criticism. That, however, was not the custom of the Royal Aeronautical Society, and he hoped that anybody in the audience who had any remarks to make of any sort or kind would not hesitate to make them.

Mr. BREWER said he would first like to draw attention to a phase in Langley's character which had only been touched upon very lightly by the lecturer, and that was the great courage which Langley showed in professing his belief in the possibility of mechanical flight at a time when anyone who professed that belief was regarded as a crank. Some of them could remember that time, and some of them had forgotten it. To show how strongly that feeling existed, he would point out that the United States Patent Office refuses at the present time to grant patents for perpetual motion, because they believed perpetual motion to be impossible. In Langley's day patents for flying machines were refused for the same reason, so they could imagine the courage that was required in a man who had already established his reputation in another field of science publicly to proclaim his belief in the possibility of flight and to devote the remainder of his life to proving that possibility. That was no small service that he did for aviation. He helped those who came after him by his example, and he would always be loved and remembered for it. There had been other prophets before Langley. There had been some Englishmen before who made models and prophesied. Henson, for instance, prophesied mechanical flight, and he made models as well. He made a model of a monoplane with two propellers to be driven by steam, but he accomplished no flights with these models as far as he (the speaker) was aware. Hargraves, in Australia, made power-driven models which flew before the time when Langley made his models fly. They were smaller than Langley's models, and they did not fly so far as Langley's models; but

still they were power-driven models driven by compressed air engines, and they were actually flying machines on a small scale. That, however, did not detract from the work of Langley because Langley's models were so much larger; but it did establish the fact that Langley's models were not the first power-driven models or the first power-driven mechanisms to fly. In the second paragraph of the lecture, Prof. Bairstow said:—"The end of this period—Langley's experiments from 1897 to 1903—is significant, coinciding almost precisely with the earliest successes of the Wright Brothers," and he would like to say a few words on that paragraph because Mr. Orville Wright was not present to reply to it. In what way could this period be significant, as stated by the lecturer? Prof. Bairstow told them that Prof. Langley published his work in two or three volumes. To be precise, Prof. Langley published a book which he entitled "Experiments in Aerodynamics" in 1891. His second book was a reprint of a paper read before the Aeronautical Congress at Chicago in 1893, and the third volume—a copy of which was on the table that evening—was the Langley Memoirs, published in 1911. Between 1893 and 1903, when the Wright Brothers flew, Langley published no data of his work. All his work was done quietly and, so far as he was aware, there was no publication of any of that work in that period of ten years. What was significant, then? Prof. Bairstow suggested that there was something significant in the Wrights flying ten years after these first two publications, and he could only imagine that the significance suggested was that the Wrights had obtained some information from Langley during the period when he was quietly trying his models and attempting to be the first to fly a full-size machine. He was strengthened in that reading of what Prof. Bairstow had said because he also stated that Langley helped the Wrights. So far as he personally knew, the Wrights obtained no information at all from Langley. The Wrights were entirely independent experimenters. They knew of the two earlier books of Langley, but there was nothing in their work which suggested that anything in these books had been of any value to them. He was very glad that Prof. Bairstow disowned the Hammondsport trials. The paper which he—the speaker—read before the Society last year on the Hammondsport trials involved a certain amount of risk. He had to show that high officials of the Smithsonian Institution had committed a fraud on the public. Two or three months ago he visited the Smithsonian Institution at Washington and saw the Langley machine hanging up with the same untrue label on it, saying that it was Langley's machine and that it had been flown at Hammondsport, and he was glad that Prof. Bairstow discountenanced those trials which were made in 1914. The attempt to prove that the Langley full-size machine of 1903 had been flown in the war period was untrue; the Langley machine had never been flown. Prof. Bairstow had stated in his Wilbur Wright lecture three years previously that the Langley machine had been flown in the war period, and he had good justification for making that statement, because it was based on the official report of the Smithsonian Institution. But that official report was untrue. As they knew, Prof. Langley was at one time secretary of the Smithsonian Institution, and Dr. Walcott was the present secretary.

Professor Bairstow had referred to the early flights of the Wright Brothers as secret flights. On the first day when the Wrights first flew into the air they sent invitations to everybody within six miles to see the flights. It was in December, and there were cold winds and nobody came. The lighthouse keeper and two or three other men in the actual locality came, so those first flights were not secret. Take the flights of the following year—in the early part of 1904 at Dayton. He knew the flying ground at Dayton because it was on the Wrights' flying ground that he was taught to fly. He took his ticket on the Wrights' ground at Dayton. A main road ran alongside down one boundary, a by-road adjoined another side of the field, the main railway which crosses the American Continent was within 200 yards, and electric cars ran between the railway and the main road. On the day when they were ready for their first flights

in 1904, the Wrights sent invitations to every newspaper to come and see the flights. Twelve or more representatives of newspapers were there, and about 50 other people also turned up. The engine would not work, and the flights for that day were a failure. No flight was made. The reporters came out the next day, but something else happened and no flights could be made. The reporters, although they said nothing, thought there was no use in coming out on further occasions. When the Wrights were flying for several minutes at a time the reporters heard of it, but they could not distinguish between airships and aeroplanes. In fact, they were all airships to the reporters, and they knew that Santos Dumont could fly for more than two or three minutes, and so they were very little interested. But he could not see how these flights could be regarded as secret flights with people passing in the trains and farmers on the other side of the hedge, and anyone could see them. Therefore it was unfair to describe the early flights of the Wright Brothers as secret flights.

At this point the CHAIRMAN asked Mr. Brewer to bring his remarks to a close as soon as possible owing to the lateness of the hour.

In conclusion, Mr. BREWER suggested that anyone who wished to follow up Langley's work should read two leading articles in "Nature"—one on November 3rd, 1921, and the other on January 26th, 1922, together with his (the speaker's) reply to those articles on March 9th this year. He had always imagined that Prof. Bairstow was the writer of those articles, and perhaps he would contradict him if he were wrong.

Prof. BAIRSTOW, replying to Mr. Brewer, said that the writing and preparation of the lecture was to some extent a consequence of the reading of Mr. Brewer's paper on the Hammondsport trials. He was also quite ready to remove any doubt there might be by saying that the "Nature" articles were written by himself. Unfortunately, Mr. Brewer and himself looked upon some of these points very differently. He would like to make it as clear as he possibly could that he regarded the Wright Brothers as the first flyers and the first producers of man-carrying aeroplanes. There was no doubt about it that Langley failed and failed after actual attempt at demonstration. He could not, however, go quite so far as Mr. Brewer did when he said the Wright Brothers received no assistance from Langley. He himself did not mean in papers, nor in curves, nor even his published books; but the fact that there was a man in their own country who professed belief—a belief to which Mr. Brewer himself had paid tribute—in the possibility of flying, and who had in 1896 demonstrated this by flying models, could not fail to place the Wrights under obligation to Langley. They must have received assistance from him in the sense which he had tried to indicate in the concluding remarks of the paper. It was not the data that Langley provided; it was the inspiration and his general idea, and he really believed that Mr. Brewer and himself were on common ground in this matter.

There was another point on which they differed a little also, although not seriously, and that was as to the secrecy of the flights made by the Wright Brothers. He himself was not specially interested in aviation in 1903. His interest arose when they heard of flights being carried out in France, and he remembered quite well that in the period between 1903 and 1908 it was so doubtful as to whether the Wrights had flown that many people in this country did not know of it at all. In fact, it was openly doubted here. Whether the flights ought to be described as secret or not was not a point which he desired to stress, but the fact was that very little was known of the Wright aeroplane until it was patented and had become a commercial proposition. That, he believed, was what Langley meant when he said the next stage was the commercial stage and was left for others. The Wright Brothers had done excellent work which he did not think detracted from Langley's.

The proceedings then closed.

# HELICOPTERS.

BY JOHN CASE, M.A., F.R.AË.S.

(*Continued.*)

## PERFORMANCE CALCULATIONS.

17. At present, while we have but a vague conception of the form a practical helicopter is likely to take, it is useless to attempt to set out in detail the dynamics of helicopter flight or performance calculations. For the evolution of a practical machine of this type it seems to the author that it will be necessary first to study the stability equations neglecting everything but gravity and the screws themselves. This should show what stabilising surfaces are required, and it is with this object that the stability equations are given below. In the meanwhile it is perhaps worth while to obtain a broad view of the ground to be surveyed, and we must expect the work to be more complicated than in the case of the aeroplane.

For the present we shall suppose that there are no auxiliary tractor screws and that forward motion is obtained by inclining the axes of the supporting screws. In this case the centre of gravity of the whole machine is not fixed relative to the screws or body, which at once introduces complication to the equations of steady motion; the position of the centre of gravity will be a function of the inclination of the airscrew axes.

The properties of an aerofoil are completely determined by three absolute coefficients which are functions of a single variable, namely, the angle of incidence. Similarly, the properties of a helicopter screw can be defined by absolute coefficients, but they are six in number and each is a function of three variables. The six coefficients are those giving the forces along, and the couples about, three axes of reference; the independent variables are $V/nD$, the inclination of the axis to the direction of motion of the centre, and an angle defining the inclination of the blade sections to the plane of rotation. This last is not merely the pitch angle of the section, but the angle through which the blades are turned, voluntarily or automatically, since we must pre-suppose that the airscrews are of variable pitch.

With regard to the six coefficients, these may be taken as:—

(i.) A force coefficient along the axis of the screw, denoted by $k_T$.
(ii.) A force coefficient parallel to the axis of $z$ (Fig. 11).
(iii.) A force coefficient parallel to the axis of $y$ (Fig. 11); this is zero.
(iv.) A torque coefficient ($k_Q$) about the axis of the screw.
(v.) A coefficient defining the mean distance of the line of action of the force $Z$ (§ 10) from the centre of the screw.
(vi.) A similar coefficient for the force $Y$, and this is zero.

Thus the number of coefficients required completely to define the properties of the screws reduces to four. In place of (i.) and (ii.) above we may take coefficients of forces along and normal to the flight path, *i.e.*, to the direction of the relative wind. It must remain to be seen which procedure is the more convenient.

For each position of the blades of a variable pitch screw, or for each screw of fixed pitch, each of these coefficients will be presented as a family of curves plotted on a base of $V/nD$, each member of the family corresponding with a given inclination of the axis of the screw.

18. For climbing vertically we can, from the data of the screws and the drawings of the machine, prepare curves such as shown in Fig. 17. From these

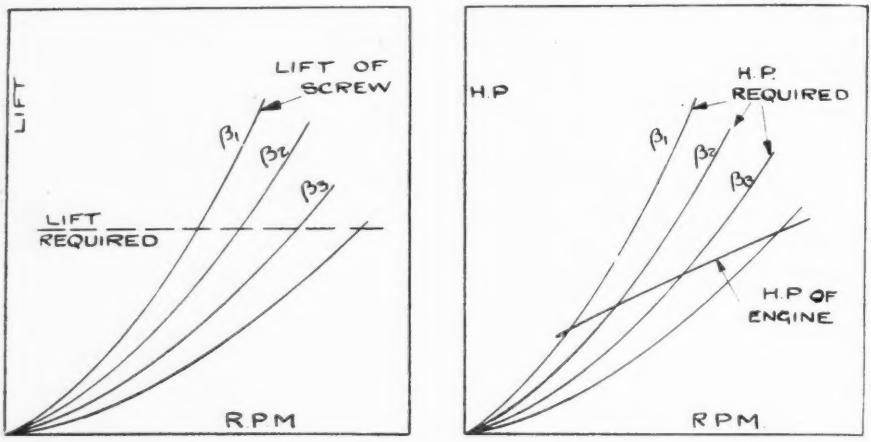



FIG. 17.

we can obtain the maximum rate of climb for a given weight, and the maximum weight for a given rate of climb, etc.

19. For climbing in an inclined path the conditions are more complicated, since more variables are introduced. In order to be consistent with the system of axes chosen for forming the stability equations, we shall suppose the axis of $x$ to be in the direction of motion, *i.e.*, through the c.g. of the whole machine and tangential to the flight path; the axis of $z$ is along the normal to the flight path (see Fig. 18).

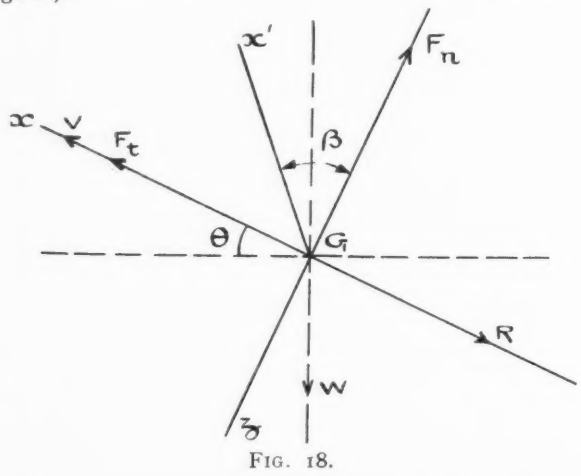



FIG. 18.

In Fig. 18, $Gx'$ is the axis of the airscrew* and $Gx$ is the direction of motion.

* Or the line of action of the resultant thrust if there be several airscrews.

Let $F_n$ and $F_t$ be the forces along the tangent and normal to the flight path. Then let

$$F_n = \rho n^2 D^4 k_n = \rho n^2 D^4 f_n (V/nD, \beta)$$
$$F_t = \rho n^2 D^4 k_t = \rho n^2 D^4 f_t (V/nD, \beta)$$

Then $k_n$ and $k_t$ correspond with $k_L$ and $k_D$ for an aerofoil and are functions of $V/nD$ and $\beta$.

Let $R$ be the resistance of the whole of the machine minus the screws.

Then in steady motion we must have:—

$$F_n = W \cos\theta \quad . \quad . \quad . \quad . \quad . \quad (16)$$
$$F_t - R = W \sin\theta \quad . \quad . \quad . \quad . \quad . \quad (17)$$

For given values of $\beta$ we can draw curves of $F_n$ and $F_t$ against $V$ for various values of $n$. We can also draw curves of $R$ against $V$ for different values of $\theta$, as in Fig. 19.

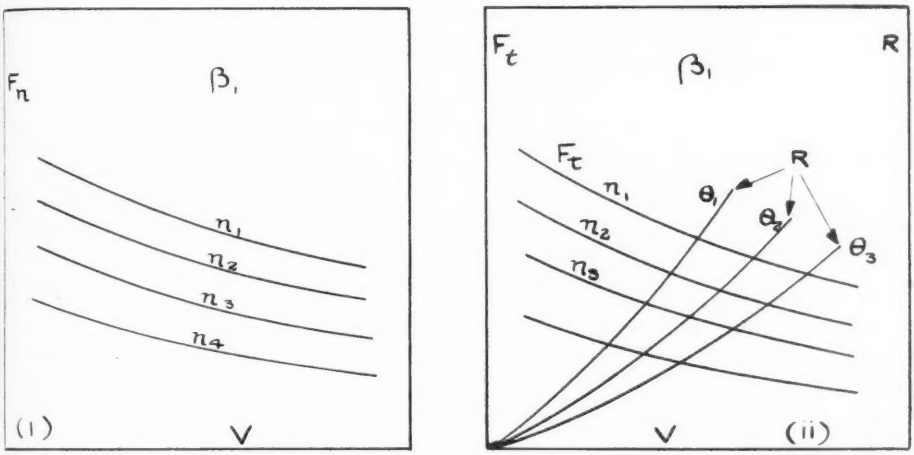



FIG. 19.

Then, taking an arbitrary value of $\theta$, we find from Fig. 19 (i.) an infinite number of pairs of values of $n$ and $V$ which satisfy equation (16), and by Fig. 19 (ii.) we can test if equation (17) is satisfied; by a succession of trials we find values of $V$, $n$ and $\theta$, which satisfy both equations (16) and (17). By this means we can draw curves of $V$ against $\theta$ for various values of $n$. This process can be repeated for successive values of $\beta$. Finally taking into consideration the power required by the screws and the power available, we can draw curves of the maximum $V$ possible, for different values of $\beta$, on a base of $\theta$.

These remarks should be sufficient to indicate the nature of the calculations which will be involved, but, as we have remarked above, it would not be profitable to go into any greater detail at the present stage.

## STABILITY.

20. A complete general treatment of the problem of helicopter stability is hardly possible with our present knowledge of the properties of airscrews moving with large sideslip velocities, but it is hoped that the following notes will be of interest and that they will be sufficient to indicate the lines along which the work must proceed. The equations given below should be sufficient for the examina-

tion of the stability when the motion is nearly vertical, ascending or descending with or without the engines, besides showing the nature of the general stability equation.

In what follows we shall limit ourselves to the consideration of rectilinear motion, and from the equations given here it will be easy to write down by analogy the equations for curvilinear motion.

Except where stated otherwise the notation is the standard stability notation as used for aeroplanes.

**21. Axes.**

The origin is at the centre of gravity $G$ of the helicopter and the axes are fixed in the machine, which we assume to have at least one plane of symmetry. The axis of $x$ is in the direction of the undisturbed motion; the axis of $z$ is in the plane of symmetry, perpendicular to the axis of $x$, and in such a direction that when $Gx$ is horizontal $Gz$ is downwards; the axis of $y$ is perpendicular to $Gx$ and $Gz$ and forms with them a system of right-handed axes. The axis of $x$ makes an angle $\theta_0$ with the horizon in the undisturbed motion, and the velocity of the machine in the direction $Gx$ is $V$.

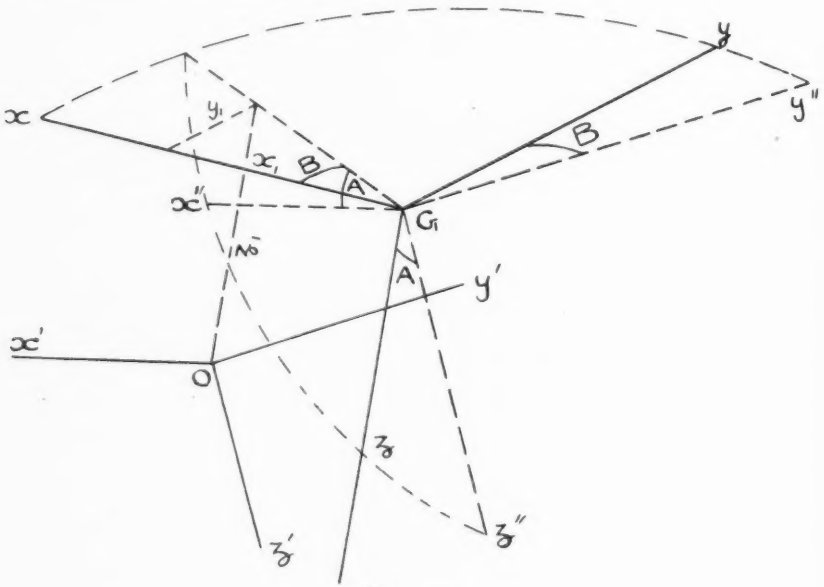



FIG. 20.

A second system of axes has its origin at $O$, the centre of one of the screws, and the co-ordinates of $O$ referred to $G$ are $(x_1, y_1, z_1)$. $Ox'$ is the axis of the screw; it makes an angle $A$ with the plane $xGy$ and lies in a plane making an angle $B$ with the plane $xGz$. Usually $z_1$ and $A$ will be negative.

With the hope of making clear the relationship of the two systems a third set of axes, $Gx''$, $Gy''$, $Gz''$, is shown dotted; these are parallel to $Ox'$, $Oy'$, $Oz'$, respectively.

All forces, moments, velocities and derivatives when referred to the helicopter axes $(Gxyz)$ will be denoted by unaccented letters; when referred to the airscrew axes $(Ox'y'z')$ they will be denoted by the corresponding accented letters.

## 22. Relations Between the Two Systems of Axes.

If $u$, $v$, $w$, $p$, $q$, $r$ denote velocities referred to the helicopter axes the corresponding velocities referred to the airscrew axes are:—

$$\left.\begin{aligned} u' &= +\bar{u}\cos A\cos B + \bar{v}\cos A\sin B + \bar{w}\sin A \\ v' &= -\bar{u}\sin B + v\cos B \\ w' &= -\bar{u}\sin A\cos B - \bar{v}\sin A\sin B + w\cos A \\ p' &= p\cos A\cos B + q\cos A\sin B + r\sin A \\ q' &= -p\sin B + q\cos B \\ r' &= -p\sin A\cos B - q\sin A\sin B + r\cos A \end{aligned}\right\} \quad . \quad . \quad (20)$$

where

$$\left.\begin{aligned} \bar{u} &= u - y_1 r + z_1 q \ . \ . \ . \\ \bar{v} &= v - z_1 p + x_1 r \ . \ . \ . \\ \bar{w} &= w - x_1 q + y_1 p \ . \ . \ . \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (21)$$

Again, if $mX$, $mY$, $mZ$, $L$, $M$, $N$, are the forces and couples referred to the helicopter axes, and accented letters refer to the airscrew axes, we have

$$\left.\begin{aligned} X &= X'\cos A\cos B - Y'\sin B - Z'\sin A\cos B \\ Y &= X'\cos A\sin B + Y'\cos B - Z'\sin A\sin B \\ Z &= X'\sin A + Z'\cos A \\ L &= L'\cos A\cos B - M'\sin B - N'\sin A + mZy_1 - mYz_1 \\ M &= L'\cos A\sin B + M'\cos B + mXz_1 - mZx_1 \\ N &= L'\sin A + N'\cos A + mYx_1 - mXy_1 \end{aligned}\right\} \quad . \quad . \quad (22)$$

## 23. The Torque Equation.

In general the forces and couples acting on the helicopter will depend on the speed of rotation of the screws as well as the six velocity components, $u$, $v$, $w$, $p$, $q$, $r$, so that there will be resistance derivatives with respect to seven variables instead of six. It will also be necessary to introduce the torque equation for the screws. Thus, the general stability equation will be a seven-row determinant equated to zero.

The extra equation is found as follows:—

Let $I$ = the moment of inertia of a screw.
$Q'_a$ = the torque on the airscrew due to the air reactions.
$Q_e$ = the driving torque due to the engine.
$n$ = the speed of rotation of the screw.

Then, denoting the derivatives by suffixes in the usual way, the equation for the screw is:—

$$-2\pi In = Q'_{au}u' + Q'_{av}v' + Q'_{aw}w' + Q'_{ap}p' + Q'_{aq}q' + Q'_{ar}r' + (Q'_{an} - Q_{en})n \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or it may be written

$$-2\pi In = Q_{au}u + Q_{av}v + Q_{aw}w + Q_{ap}p + Q_{aq}q + Q_{ar}r + (Q_{an} - Q_{en})n \quad . \quad . \quad . \quad . \quad . \quad (18a)$$

## 24. Stability Equation.

Postponing for the present the evaluation of the derivatives, we can now write down the general stability equation:—

$$\begin{vmatrix} X_u - \lambda & X_v & X_w & X_p & \lambda X_q - g\cos\theta_o & X_r & X_n \\ Y_u & Y_v - \lambda & Y_w & \lambda Y_p + g\cos\theta_o & Y_q & \lambda(Y_r - V) + g\sin\theta_o & Y_n \\ Z_u & Z_v & Z_w - \lambda & Z_p & \lambda(V + Z_q) - g\sin\theta_o & Z_r & Z_n \\ L_u & L_v & L_w & \lambda L_p - A\lambda^2 & L_q & \lambda L_r + E\lambda^2 & L_n \\ M_u & M_v & M_w & M_p & \lambda M_q - B\lambda^2 & M_r & M_n \\ N_u & N_v & N_w & \lambda N_p + E\lambda^2 & N_q & \lambda N_r - C\lambda^2 & N_n \\ Q_{au} & Q_{av} & Q_{aw} & Q_{ap} & Q_{aq} & Q_{ar} & 2\pi I\lambda + Q_{an} - Q_{en} \end{vmatrix} = 0 \quad . \quad . \quad . \quad (19)$$

This is the general equation for the stability of the motion of a helicopter in a straight line; it will be seen to be of the tenth degree. The equation for curvilinear motion can be written down at once by comparing this with equation (72) on p. 490 of Bairstow's "Applied Aerodynamics."

The derivatives will have contributions from each of the screws and from the rest of the machine, and these must be added together. We shall confine our attention here to those parts of the derivatives which are due to the screws.

We have not attempted the expansion of the above determinant as it seems that no good purpose would be served by it at present; we shall see instead what simplifications can be made in special cases.

### 25. Formation of the Derivatives.

When the direction of motion makes a small angle with the axis of the screw the derivatives can be calculated by the formulæ given by H. Glauert in R. and M. 642 and reproduced here in Table VI., but at present we have no formulæ for their evaluation when the angle between the direction of motion and the axis of the screw is large. Perhaps we might derive formulæ for this case by Mr. Glauert's method, but it seems likely that it will be necessary to do a considerable amount of work in the wind tunnel before we can satisfactorily study the general stability of a helicopter. This means that for the present we are limited to motion which is nearly vertical.

### 26. Expressions for the Derivatives.

Consider as an example the $X$ derivatives. If there be a disturbance denoted by $(u, v \ldots r)$ referred to the helicopter axes or $(u', v' \ldots r')$ referred to the airscrew axes, the change effected in $X$ is given by

$$dX = dX' \cos A \cos B - dY' \sin B - dZ' \sin A \cos B \quad . \quad . \quad (23)$$

by (22). Now

$$dX' = X'_u u' + X'_v v' + \ldots + X'_r r' \quad . \quad . \quad . \quad (24)$$

with similar expressions for $dY'$ and $dZ'$.

We can substitute for $u', v' \ldots r'$ in terms of $u, v \ldots r$ by means of (20) and (21), thus expressing $dX'$, $dY'$ and $dZ'$ in terms of $X'_u \ldots X'_r$ and $u' \ldots r'$. We then substitute in (23) and the result is of the form

$$dX = f(X'_u \ldots X'_r;\ Y'_u \ldots Y'_r;\ Z'_u \ldots Z'_r;\ u,\ v \ldots r).$$

Then $X_u$ is the coefficient of $u$ in this expression, and so on.

By this means the derivatives for the helicopter are expressed in terms of those for the screw. Formulæ for calculating the latter, when the velocity of sideslip is small compared with the velocity along the axis of the screw, are given below in Table VI.

**TABLE VI.**

Airscrew derivatives referred to axes through centre of airscrew (Glauert in R. and M. 642).

| | $u$ | $v$ | $w$ | $p$ | $q$ | $r$ |
|---|---|---|---|---|---|---|
| $mX'$ | $\frac{2T}{V} \cdot \frac{\lambda_T(1+\lambda_P)-\lambda_Q}{1+\lambda_P-\lambda_Q}$ | 0 | 0 | $\frac{DTJ}{\pi V} \cdot \frac{\lambda_P(1-\lambda_T)}{1+\lambda_P-\lambda_Q}$ | 0 | 0 |
| $mY'$ | 0 | $-\frac{k_1QJ(1-\lambda_Q)}{2\pi VD}$* | $\frac{k_1QJ(1-\lambda_Q)}{2\pi VD}$† | 0 | $\frac{Q}{V}\lambda_Q$* | $-\frac{Q}{\Gamma}\lambda_Q$† |
| $mZ'$ | 0 | $\frac{k_1QJ(1-\lambda_Q)}{2\pi VD}$† | $-\frac{k_1QJ(1-\lambda_Q)}{2\pi VD}$§ | 0 | $-\frac{Q}{\Gamma}\lambda_Q$† | $\frac{Q}{V}\lambda_Q$§ |
| $L'$ | $-\frac{2Q}{V} \cdot \frac{\lambda_P\lambda_Q}{1+\lambda_P-\lambda_Q}$ | 0 | 0 | $-\frac{DQJ}{\pi V} \cdot \frac{\lambda_P(1-\lambda_Q)}{1+\lambda_P-\lambda_Q}$ | 0 | 0 |
| $M'$ | 0 | $\frac{-DTJ}{2\pi V}(1-\lambda_T)$* | $\frac{DTJ(1-\lambda_T)}{2\pi V}$† | 0 | $k_2D^2\frac{T}{V}\lambda_T$* | $-k_2D^2\frac{T}{\Gamma}\lambda_T$† |
| $N'$ | 0 | $\frac{DTJ(1-\lambda_T)}{2\pi V}$† | $-\frac{DTJ(1-\lambda_T)}{2\pi V}$§ | 0 | $-k_2D^2\frac{T}{\Gamma}\lambda_T$† | $k_2D^2\frac{T}{\Gamma}\lambda_T$§ |

When the propeller has only two blades, expressions marked * must be multiplied by $(1-\cos 2\pi nt)$, those marked † by $\sin 2\pi nt$, and those marked § by $(1+\cos 2\pi nt)$.

When there are more than two blades the terms marked † are zero.

The above refer to a right-handed screw; for a left-handed screw the sign of the following must be changed:—

$$Y'_w\ Z'_v\ L'_u\ M'_v\ N'_w$$
$$X'_p\ Y'_q\ Z'_r\ M'_r\ N'_q$$

Also $\lambda_T = \frac{1}{2}\frac{J}{k_T}\cdot\frac{dk_T}{dJ}$

$\lambda_Q = \frac{1}{2}\frac{J}{k_Q}\cdot\frac{dk_Q}{dJ}$

$k_1$ may be taken as 14.4

$k_2$ ,, ,, 0.10

$2\lambda_P = 1 - \frac{n}{P}\frac{dP}{dn}$ where $P$ is the power of the engine.

It will be seen that in this case many of the derivatives referred to the air-screw axis disappear, which results in a corresponding simplification of the equations.

The derivatives for the screw referred to the main axes are in this case:—

$$X_u = X'_u \cos^2 A \cos^2 B + Y'_v \sin^2 B + Y'_w \sin A \sin B \cos B + Z'_v \sin A \sin B \cos B + Z'_w \sin^2 A \cos^2 B.$$

$$X_v = X'_u \cos^2 A \sin B \cos B - Y'_v \sin B \cos B + Y'_w \sin A \sin^2 B - Z'_v \sin A \cos^2 B + Z'_w \sin^2 A \sin B \cos B.$$

$$X_w = X'_u \sin A \cos A \cos B - Y'_w \cos A \sin B - Z'_w \sin A \cos A \cos B.$$

$$X_p = X'_u (y_1 \sin A - z_1 \cos A \sin B) \cos A \cos B + X'_p \cos^2 A \cos^2 B. + Y'_v z_1 \cos B \sin B - Y'_w (y_1 \cos A + z_1 \sin A \sin B) \sin B. + Y'_q \sin^2 B + Y'_r \sin A \sin B \cos B. + Z'_v z_1 \sin A \cos^2 B - Z'_w (y_1 \cos A + z_1 \sin A \sin B) \sin A \cos B. - Z'_q \sin A \sin B \cos B + Z'_r \sin^2 A \cos^2 B.$$

$$X_q = X'_u (z_1 \cos A \cos B - x_1 \sin A) \cos A \cos B + X'_p \cos^2 A \sin B \cos B. + Y'_v z_1 \sin^2 B + Y'_w (z_1 \sin A \cos B + x_1 \cos A) \sin B. + Y'_q \sin B \cos B - Y'_r \sin A \sin^2 B + Z'_v z_1 \sin A \sin B \cos B. + Z'_w (z_1 \sin A \cos B + x_1 \cos A) \sin A \cos B - Z'_q \sin A \cos^2 B. + Z'_r \sin^2 A \sin B \cos B.$$

$$X_r = X'_u (x_1 \sin B - y_1 \cos B) \cos^2 A \cos B + X'_p \sin A \cos A \cos B. - Y'_v (x_1 \cos B + y_1 \sin B) \sin B - Y'_w (x_1 \sin B - y_1 \cos B) \sin A \sin B. + Y'_r \cos A \sin B - Z'_v (x_1 \cos B + y_1 \sin B) \sin A \cos B. + Z'_w (x_1 \sin B - y_1 \cos B) \sin^2 A \cos B - Z'_r \sin A \cos A \cos B.$$

$$Y_u = X'_u \cos^2 A \sin B \cos B - Y'_v \sin B \cos B - Y'_w \sin A \cos^2 B + Z'_v \sin A \sin^2 B + Z'_w \sin^2 A \sin B \cos B.$$

$$Y_v = X'_u \cos^2 A \sin^2 B + Y'_v \cos^2 B - Y'_w \sin A \sin B \cos B - Z'_v \sin A \sin B \cos B + Z'_w \sin^2 A \sin^2 B.$$

$$Y_w = X'_u \sin A \cos A \sin B + Y'_w \cos A \cos B - Z'_w \sin A \cos A \sin B.$$

$$Y_p = - X'_u (z_1 \cos A \sin B - y_1 \sin A) \cos A \sin B. + X'_p \cos^2 A \sin B \cos B - Y'_v z_1 \cos^2 B. + Y'_w (y_1 \cos A + z_1 \sin A \sin B) \cos B - Y'_q \sin B \cos B. - Y'_r \sin A \cos^2 B - Z'_w (y_1 \cos A + z_1 \sin A \sin B) \sin A \sin B. + Z'_q \sin A \sin^2 B + Z'_r \sin^2 A \sin B \cos B + Z'_v z_1 \sin A \sin B \cos B.$$

$$Y_q = X'_u (z_1 \cos A \cos B - x_1 \sin A) \cos A \sin B. + X'_p \cos^2 A \sin^2 B - Y'_v z_1 \sin B \cos B. - Y'_w (z_1 \sin A \cos B + x_1 \cos A) \cos B + Y'_q \cos^2 B. - Y'_r \sin A \cos^2 B + Z'_v z_1 \sin A \sin^2 B. + Z'_w (z_1 \sin A \cos B + x_1 \cos A) \sin A \sin B - Z'_q \sin A \sin B \cos B. + Z'_r \sin^2 A \sin B \cos B.$$

$$Y_r = X'_u (x_1 \sin B - y_1 \cos B) \cos^2 A \sin B + X'_p \sin A \cos A \sin B. + Y'_v (x_1 \cos B + y_1 \sin B) \cos B. - Y'_w (x_1 \sin B - y_1 \cos B) \sin A \cos B + Y'_r \cos A \cos B. - Z'_v (x_1 \cos B + y_1 \sin B) \sin A \sin B + Z'_w (x_1 \sin B - y_1 \cos B) \sin^2 A \sin B. - Z'_r \sin A \cos A \sin B.$$

$$Z_u = X'_u \sin A \cos A \cos B - Z'_v \cos A \sin B - Z'_w \sin A \cos A \cos B.$$

$$Z_v = X'_u \sin A \cos A \sin B + Z'_v \cos A \cos B - Z'_w \sin A \cos A \sin B.$$

$$Z_w = X'_u \sin^2 A + Z'_w \cos^2 A.$$

$$Z_p = - X'_u (z_1 \cos A \sin B - y_1 \sin A) \sin A + X'_p \sin A \cos A \cos B. - Z'_v z_1 \cos A \cos B + Z'_w (y_1 \cos A + z_1 \sin A \sin B) \cos A. - Z'_q \cos A \sin B - Z'_r \sin A \cos A \cos B.$$

$$Z_q = X'_u (z_1 \cos A \cos B - x_1 \sin A) \sin A + X'_p \sin A \cos A \sin B. - Z'_v z_1 \cos A \sin B - Z'_w (z_1 \sin A \cos B + x_1 \cos A) \cos A. + Z'_q \cos A \cos B - Z'_r \sin A \cos A \sin B.$$

$$Z_r = X'_u (x_1 \sin B - y_1 \cos B) \sin A \cos A + X'_p \sin^2 A. + Z'_v (y_1 \sin B + x_1 \cos B) \cos A. - Z'_w (x_1 \sin B - y_1 \cos B) \sin A \cos A + Z'_r \cos^2 A.$$

$$
\begin{aligned}
L_u = {} & + mX'_u\,(y_1 \sin A - z_1 \cos A \sin B)\cos A \cos B.\\
& - mY'_v\, z_1 \cos B \sin B - mY'_w\, z_1 \sin A \cos^2 B.\\
& - mZ'_v\, y_1 \sin A \sin B - mZ'_w\, y_1 \sin^2 A \cos B + L'_u \cos^2 A \cos^2 B.\\
& + M'_v \sin^2 B + M'_w \sin A \sin B \cos B + N'_v \sin A \sin B.\\
& + N'_w \sin^2 A \cos B.\\
L_v = {} & + mX'_u\,(y_1 \sin A - z_1 \cos A \sin B)\cos A \sin B.\\
& + mY'_v\, z_1 \cos^2 B - mY'_w\, z_1 \sin A \sin B \cos B.\\
& + mZ'_v\, y_1 \sin A \cos B - mZ'_w\, y_1 \sin^2 A \sin B.\\
& + L'_u \cos^2 A \sin B \cos B - M'_v \sin B \cos B + M'_w \sin A \sin^2 B.\\
& - N'_v \sin A \cos B + N'_w \sin^2 A \sin B.\\
L_w = {} & mX'_u\,(y_1 \sin A - z_1 \cos A \sin B)\sin A.\\
& + mY'_w\,(z_1 \cos B)\cos A + mZ'_w\, y_1 \sin A \cos A.\\
& + L'_u \sin A \cos A \cos B - M'_w \cos A \sin B - N'_w \sin A \cos A.\\
L_p = {} & mX'_u\,(-z_1 \cos A \sin B + y_1 \sin A)^2 \cos A.\\
& + mX'_p\,(y_1 \sin A - z_1 \sin B \cos A)\cos A \cos B.\\
& - mY'_v\, z_1^2 \cos^2 B.\\
& + mY'_w\,(z_1 \cos B)(z_1 \sin A \sin B + y_1 \cos A).\\
& - mY'_q\, z_1 \cos B \sin B - mY'_r\, z_1 \cos B \sin A \cos B.\\
& - mZ'_v\, y_1\, z_1 \sin A \cos B + mZ'_w\, y_1 \sin A\,(z_1 \sin A \sin B + y_1 \cos A).\\
& - mZ'_q\, y_1 \sin A \sin B - mZ'_r \sin^2 A \cos B.\\
& + L'_u\,(-z_1 \cos A \sin B + y_1 \sin A)\cos A \cos B + L'_p \cos^2 A \cos^2 B.\\
& + M'_v\, z_1 \sin B \cos B - M'_w\,(z_1 \sin A \sin B + y_1 \cos A)\sin B.\\
& + M'_q \sin^2 B + M'_r \sin A \sin B \cos B + N'_v\, z_1 \sin A \cos B.\\
& - N'_w\,(z_1 \sin A \sin B + y_1 \cos A)\sin A + N'_q \sin A \sin B.\\
& + N'_r \sin^2 A \cos B.\\
L_q = {} & + mX'_u\,(y_1 \sin A - z_1 \cos A \sin B)\,(z_1 \cos A \sin B - x \sin A).\\
& + mX'_p\,(\sin A - z_1 \cos A \sin B)\cos A \sin B.\\
& - mY'_v\, z_1^2 \cos B \sin B.\\
& - mY'_w\,(z_1 \cos B)(z_1 \sin A \cos B + x_1 \cos A).\\
& - mY'_q\, z_1 \cos^2 B - mY'_r\, z_1 \cos B \sin A \sin B.\\
& - mZ'_v\, y_1\, z_1 \sin A \sin B - mZ'_w\, y_1 \sin A\,(z_1 \sin A \cos B + x_1 \cos A).\\
& + mZ'_q\, y_1 \sin A \cos B - mZ'_r\, y_1 \sin^2 A \sin B.\\
& + L'_u\,(z_1 \cos A \sin B - x_1 \sin A)\cos A \cos B + L'_p \cos^2 A \sin B \cos B.\\
& + M'_v\, z_1 \sin^2 B + M'_w\,(z_1 \sin A \cos B + x_1 \cos A)\sin B.\\
& - M'_q \sin B \cos B + M'_r \sin A \sin^2 B + N'_v\, z_1 \sin A \sin B.\\
& - N'_w\,(z_1 \sin A \cos B + x_1 \cos A)\sin A - N'_q \sin A \cos B.\\
& + N'_r \sin^2 A \sin B.\\
L_r = {} & + mX'_u\,(y_1 \sin A - z_1 \cos A \sin B)\,(x_1 \sin B - y_1 \cos B)\cos A.\\
& + mX'_p\,(y_1 \sin A - z_1 \cos A \sin B)\sin A.\\
& + mY'_v\, z_1 \cos B\,(x_1 \cos B + y_1 \sin B).\\
& - mY'_w\, z_1 \cos B\,(x_1 \sin B - y_1 \cos B)\sin A.\\
& + mY'_r\, z_1 \cos B \cos A + mZ'_v\, y_1 \sin A\,(x_1 \cos B + y_1 \sin B).\\
& - mZ'_w\, y_1 \sin^2 A\,(x_1 \sin B - y_1 \cos B) + mZ'_r\, y_1 \sin A \cos A.\\
& + L'_u\,(x_1 \sin B - y_1 \cos B)\cos^2 A \cos B + L'_p \sin A \cos A \cos B.\\
& - M'_v\,(x_1 \cos B + y_1 \sin B)\sin B + M'_w\,(x_1 \sin B - y_1 \cos B)\sin A \sin B.\\
& - M'_r \cos A \sin B - N'_v\,(x_1 \cos B + y_1 \sin B)\sin A.\\
& + N'_w\,(x_1 \sin B - y_1 \cos B)\sin^2 A - N'_r \sin A \cos A.\\
M_u = {} & - mX'_u \cos A \cos B\,(x_1 \sin A - z_1 \cos A \cos B).\\
& + mY'_v\, z_1 \sin^2 B + mY'_w\, z_1 \sin A \sin B \cos B.\\
& + mZ'_v \sin B\,(x_1 \cos A - z_1 \sin A \cos B) + L'_u \cos^2 A \sin B \cos B.\\
& - mZ'_w\,(z_1 \sin A + x_1 \cos A)\sin A \cos B.\\
& - M'_v \sin B \cos B - M'_w \sin A \cos^2 B.\\
M_v = {} & - mX'_u \cos A \sin B\,(x_1 \sin A - z_1 \cos A \cos B).\\
& - mY'_v\, z_1 \sin B \cos B + mY'_w\, z_1 \sin A \sin^2 B.\\
& - mZ'_v \cos B\,(x_1 \cos A - z_1 \sin A \cos B).\\
& - mZ'_w \sin A \sin B\,(x_1 \cos A - z_1 \sin A \cos B) + M'_v \cos^2 B.\\
& + L'_u \cos^2 A \sin^2 B.\\
& - M'_w \sin A \sin B \cos B.
\end{aligned}
$$

$$M_w = -mX'_u \sin A\,(x_1 \sin A - z_1 \cos A \cos B).$$
$$- mY'_w \cos A\,(x_1 \sin A - z_1 \sin B).$$
$$- mZ'_w \cos A\,(x_1 \cos A - z_1 \sin A \cos B) + L'_u \sin A \cos A \sin B.$$
$$+ M'_w \cos A \cos B.$$

$$M_p = -mX'_u\,(y_1 \sin A - z_1 \cos A \sin B)\,(x_1 \sin A - z_1 \cos A \cos B).$$
$$- mX'_p\,(x_1 \sin A - z_1 \cos A \cos B) \cos A \cos B.$$
$$+ mY'_v\, z_1^2 \sin B \cos B + mY'_w\, z_1\,(y_1 \cos A + z_1 \sin A \sin B) \sin B.$$
$$- mY'_q\, z_1 \sin^2 B - mY'_r\, z_1 \sin A \sin B \cos B.$$
$$+ mZ'_v\, z_1 \cos B\,(x_1 \cos A - z_1 \sin A \cos B).$$
$$- mZ'_w\,(y_1 \cos A + z_1 \sin A \sin B)\,(x_1 \cos A - z_1 \sin A \cos B).$$
$$+ mZ'_q \sin B\,(x_1 \cos A - z_1 \sin A \cos B).$$
$$+ mZ'_r \sin A \cos B\,(x_1 \cos A - z_1 \sin A \cos B).$$
$$+ L'_u \cos A \sin B\,(y_1 \sin A - z_1 \cos A \sin B) + L'_p \cos^2 A \sin B \cos B.$$
$$- M'_v\, z_1 \cos^2 B + M'_w \cos B\,(y_1 \cos A + z_1 \sin A \sin B).$$
$$- M'_q \sin B \cos B - M'_r \sin A \cos^2 B.$$

$$M_q = mX'_u\,(x_1 \sin A - z_1 \cos A \cos B)^2.$$
$$- mX'_p \cos A \sin B\,(x_1 \sin A - z_1 \cos A \cos B).$$
$$+ mY'_v\, z_1^2 \sin^2 B - mY'_w\, z_1 \sin B\,(x_1 \cos A + z_1 \sin A \cos B).$$
$$+ mY'_q\, z_1 \sin B \cos B - mY'_r\, z_1 \sin A \sin^2 B.$$
$$+ mZ'_v\, z_1 \sin B\,(x_1 \cos A + z_1 \sin A \cos B).$$
$$+ mZ'_w\,(x_1 \cos A - z_1 \sin A \cos B)^2.$$
$$- mZ'_q \cos B\,(x_1 \cos A - z_1 \sin A \cos B) + mZ'_r \sin A \sin B\,(x_1 \cos A - z_1 \sin A \cos B).$$
$$- L'_u \cos A \sin B\,(x_1 \sin A - z_1 \cos A \cos B) + L'_p \cos^2 A \sin^2 B.$$
$$+ M'_v\, z_1 \sin B \cos B - M'_w \cos B\,(x_1 \cos A + z_1 \sin A \cos B).$$
$$+ M'_q \cos^2 B - M'_r \sin A \sin B \cos B.$$

$$M_r = -mX'_u \cos A\,(x_1 \sin B - y_1 \cos B)\,(x_1 \sin A - z_1 \cos A \cos B).$$
$$- mX'_p \sin A\,(x_1 \sin A - z_1 \cos A \cos B).$$
$$- mY'_v\,(x_1 \cos B + y_1 \sin B)\, z_1 \sin B - mY'_w\, z_1 \sin A \sin B\,(x_1 \sin B - y_1 \cos B).$$
$$+ mY'_r\, z_1 \cos A \sin B - mZ'_v\,(x_1 \cos B + y_1 \sin B)\,(x_1 \cos A - z_1 \sin A \cos B).$$
$$+ mZ'_w\,(x_1 \sin B - y_1 \cos B)\,(x_1 \cos A - z_1 \sin A \cos B) \sin A.$$
$$- mZ'_r \cos A\,(x_1 \cos A - z_1 \sin A \cos B) + L'_u\,(x_1 \sin B - y_1 \cos B) \cos^2 A \sin B.$$
$$+ L'_p \sin A \cos A \sin B + M'_v\,(x_1 \cos B + y_1 \sin B) \cos B.$$
$$- M'_w\,(x_1 \sin B - y_1 \cos B) \sin A \cos B + M'_r \cos A \cos B.$$

$$N_u = mX'_u\,(x_1 \sin B - y_1 \cos B) \cos^2 A \cos B - mY'_v\,(x_1 \cos B + y_1 \sin B) \sin B.$$
$$- mY'_w\,(x_1 \cos B + y_1 \sin B) \sin A \cos B - mZ'_v\, y_1 \sin A \sin B \cos B.$$
$$- mZ'_w\, y_1 \sin^2 A \cos^2 B + L'_u \sin A \cos A \cos B.$$
$$- N'_v \cos A \sin B - N'_w \sin A \cos A \cos B.$$

$$N_v = mX'_u\,(x_1 \sin B - y_1 \cos B) \cos^2 A \sin B + mY'_v\,(x_1 \cos B + y_1 \sin B) \cos B.$$
$$- mY'_w\,(x_1 \cos B + y_1 \sin B) \sin A \sin B + mZ'_v\, y_1 \sin A \cos^2 B.$$
$$- mZ'_w\, y_1 \sin^2 A \sin B \cos B + L'_u \sin A \cos A \sin B.$$
$$+ N'_v \cos A \cos B - N'_w \sin A \cos A \sin B.$$

$$N_w = + mX'_u\,(x_1 \sin B - y_1 \cos B) \sin A \cos A + mY'_w\,(x_1 \cos B + y_1 \sin B) \cos A.$$
$$+ mZ'_w\, y_1 \sin A \cos A \cos B + L'_u \sin^2 A + N'_w \cos^2 A.$$

$$N_p = mX'_u \cos A\,(x_1 \sin B - y_1 \cos B)\,(y_1 \sin A - z_1 \cos A \sin B).$$
$$+ mX'_p \cos^2 A \cos B\,(x_1 \sin B - y_1 \cos B).$$
$$- mY'_v\, z_1 \cos B\,(x_1 \cos B + y_1 \sin B).$$
$$+ mY'_w\,(y_1 \cos A + z_1 \sin A \sin B)\,(x_1 \cos B + y_1 \sin B).$$
$$- mY'_q \sin B\,(x_1 \cos B + y_1 \sin B) - mY'_r \sin A \cos B\,(x_1 \cos B + y_1 \sin B).$$
$$- mZ'_v\, y_1 z_1 \sin A \cos^2 B + mZ'_w\, y_1 \sin A \cos B\,(y_1 \cos A + z_1 \sin A \sin B).$$
$$- mZ'_q\, y_1 \sin A \sin B \cos B - mZ'_r\, y_1 \sin^2 A \cos B.$$
$$+ L'_u \sin A\,(y_1 \sin A - z_1 \cos A \sin B) + L'_p \sin A \cos A \cos B.$$
$$- N'_v\, z_1 \cos A \cos B + N'_w \cos A\,(y_1 \cos A + z_1 \sin A \sin B).$$
$$- N'_q \cos A \sin B - N'_r \sin A \cos A \cos B.$$

$$
\begin{aligned}
N_q = &- mX'_u \cos A\,(x_1 \sin B - y_1 \cos B)\,(x_1 \sin A - z_1 \cos A \cos B).\\
&+ mX'_p \cos^2 A \sin B\,(x_1 \sin B - y_1 \cos B).\\
&- mY'_v\, z_1 \sin B\,(x_1 \cos B + y_1 \sin B).\\
&- mY'_w\,(x_1 \cos B + y_1 \sin B)\,(x_1 \cos A + z_1 \sin A \cos B).\\
&+ mY'_q \cos B\,(x_1 \cos B + y_1 \sin B) - mY'_r \sin A \sin B\,(x_1 \cos B + y_1 \sin B).\\
&- mZ'_v\, y_1\, z_1 \sin A \sin B \cos B + mZ'_w\, y_1 \sin A \cos B\,(x_1 \cos A + z_1 \sin A \cos B).\\
&+ mZ'_q\, y_1 \sin A \cos^2 B - mZ'_r\, y_1 \sin^2 A \sin B.\\
&- L'_u \sin A\,(x_1 \sin A - z_1 \cos A \cos B) + L'_p \sin A \cos A \sin B.\\
&- N'_v\, z_1 \cos A \sin B - N'_w \cos A\,(x_1 \cos A + z_1 \sin A \cos B).\\
&+ N'_q \cos A \cos B - N'_r \sin A \cos A \sin B.\\
N_r = &\; mX'_u \cos^2 A\,(x_1 \sin B - y_1 \cos B)^2.\\
&+ mX'_p \sin A \cos A\,(x_1 \sin B - y_1 \cos B).\\
&+ mY'_v\,(x_1 \cos B + y_1 \sin B)^2.\\
&- mY'_w\,(x_1 \cos B + y_1 \sin B)\,(x_1 \sin B - y_1 \cos B) \sin A.\\
&+ mY'_r \cos A\,(x_1 \cos B + y_1 \sin B) + mZ'_v\, y_1 \sin A \cos B\,(x_1 \cos B + y_1 \sin B).\\
&- mZ'_w \sin A \cos B\,(x_1 \sin B - y_1 \cos B) + mZ'_r\, y_1 \sin A \cos A \cos B.\\
&+ L'_u \sin A \cos A\,(x_1 \sin B - y_1 \cos B) + L'_p \sin^2 A.\\
&+ N'_v \cos A\,(x_1 \cos B + y_1 \sin B) - N'_w \sin A \cos A\,(x_1 \sin B - y_1 \cos B).\\
&+ N'_r \cos^2 A.
\end{aligned}
$$

This completes the tabulation of the derivatives of the screw referred to the helicopter axes; the accented letters are the derivatives of the screw referred to its own axes and given above.

The expressions given are perfectly general and will apply to a helicopter having any number of screws disposed on any suitable axes, provided the sideslip velocity is small compared with the axial velocity.

**27. Simplification when there are more than two blades.**

The following derivatives disappear:—

$$Y'_w \;\; Y'_r \;\; Z'_v \;\; Z'_q \;\; M'_w \;\; M'_r \;\; N'_v \;\; N'_q.$$

In view of the periodic nature of the forces when the helicopter has general motion, as shown above, we shall suppose the screws have always at least four blades.

**28. The Torque Derivatives.**

In steady motion we have

$$Q'_a = -L'$$

Then

$$Q'_{au} = -L'_u$$
$$Q'_{ap} = -L'_p$$

and in the case we are considering, *i.e.*, when the sideslip velocity is small, all the other $Q'_a$ derivatives are zero.

Then equation (18) gives

$$2\pi I\dot{n} = L'_u \,.\, u' + L'_p \,.\, p' + (Q_{en} - Q_{an})\; n.$$

To put this in the form (18*a*) we substitute for $u'$ and $p'$ from (20) and (21) and then $Q_{au}$ is the coefficient of $u$ in the resulting expression, and so on. We find:—

$$
\begin{aligned}
Q_{au} &= L'_u \cos A \cos B.\\
Q_{av} &= L'_u \cos A \sin B.\\
Q_{aw} &= L'_u \sin A.\\
Q_{ap} &= L'_u\,(y_1 \sin A - z_1 \cos A \sin B) + L'_p \cos A \cos B.\\
Q_{aq} &= L'_u\,(z_1 \cos A \cos B - x_1 \sin A) + L'_p \cos A \sin B.\\
Q_{ar} &= L'_u\,(x_1 \sin B - y_1 \cos B) \cos A + L'_p \sin A.
\end{aligned}
$$

**The n Derivatives.**

The quantities $X_n$ $Y_n$ . . . are expressed in terms of the quantities $X'_n$ $Y'_n$ . . . by means of equations (22).

29. We shall now examine some particular cases.

**I. Two Contrary-Turning Screws on a Single Axis.**

We shall have $y_1 = 0$ and $B = 0$. The derivatives reduce to :—

$X_u = X'_u \cos^2 A + Z'_w \sin^2 A.$ $X_v = 0.$
$X_w = (X'_u - Z'_w) \sin A \cos A.$ $X_p = 0.$
$X_q = X'_u (z_1 \cos A - x_1 \sin A) \cos A + Z'_w (z_1 \sin A + x_1 \cos A) \sin A.$
$X_r = 0.$
$Y_u = 0.$ $Y_v = Y'_v.$ $Y_w = 0.$
$Y_p = -z_1 Y'_v.$ $Y_q = 0.$ $Y_r = x_1 Y'_v.$
$Z_u = (X'_u - Z'_w) \sin A \cos A.$ $Z_v = 0.$
$Z_w = X'_u \sin^2 A + Z'_w \cos^2 A.$
$Z_p = 0.$ $Z_r = 0.$
$Z_q = X'_u (z_1 \cos A - x_1 \sin A) \sin A - Z'_w (z_1 \sin A + x_1 \cos A) \cos A.$
$L_u = 0.$ $L_w = 0.$
$L_v = mY'_v z_1.$
$L_p = -mY'_v z_1^2 + L'_p \cos^2 A + N'_r \sin^2 A - mZ'_r \sin^2 A.$
$L_q = -mY'_q z_1 - L'_u x_1 \sin A \cos A - N'_w (z_1 \sin A + x_1 \cos A) \sin A.$
$L_r = mY'_v x_1 z_1 - (N'_r - L'_p) \sin A \cos A.$
$M_u = mX'_u (z_1 \cos A - x_1 \sin A) \cos A - mZ'_w (z_1 \sin A + x_1 \cos A) \sin A.$
$M_v = 0.$
$M_w = mX'_u (z_1 \cos A - x_1 \sin A) \sin A - mZ'_w (z_1 \sin A + x_1 \cos A) \cos A.$
$M_p = mX'_p (z_1 \cos A - x_1 \sin A) \cos A + mZ'_r (z_1 \sin A + x_1 \cos A) \sin A - z_1 M'_v.$
$M_q = mX'_u (x_1 \sin A - z_1 \cos A)^2 + mZ'_w (x_1 \cos A + z_1 \sin A)^2 + M'_q.$
$M_r = mX'_p (z_1 \cos A - x_1 \sin A) \sin A - mZ'_r (x_1 \cos A + z_1 \sin A) \cos A + x_1 M'_v.$
$N_u = 0.$ $N_v = x_1 \cdot mY'_v.$ $N_w = 0.$
$N_p = -mY'_v z_1 x_1 - (N'_r - L'_p) \sin A \cos A.$
$N_q = mY'_q x_1 + L'_u (z_1 \cos A - x_1 \sin A) \sin A - N'_w (x_1 \cos A + z_1 \sin A) \cos A.$
$N_r = N'_r \cos^2 A + L'_p \sin^2 A + mY'_v \cdot x_1^2.$

We also have in this case :—

$Q_{au} = L'_u \cos A.$ $Q_{av} = 0.$
$Q_{aw} = L'_u \sin A.$
$Q_{ap} = L'_p \cos A.$
$Q_{aq} = L'_u (z_1 \cos A - x_1 \sin A).$
$Q_{ar} = L'_p \sin A.$
$X_n = X'_n \cos A - Z'_n \sin A.$ $Y'_n = 0.$
$Z_n = X'_n \sin A + Z'_n \cos A.$
$L_n = L'_n \cos A - N'_n \sin A.$
$M_n = -m (X_n z_1 - Z_n x_1).$
$N_n = L'_n \sin A + N'_n \cos A.$

The stability equation is now :—

$$\begin{vmatrix} X_u - \lambda & 0 & X_w & 0 & \lambda X_q - g\cos\theta_0 & 0 & X_n \\ 0 & Y_v - \lambda & 0 & \lambda Y_p + g\cos\theta_0 & 0 & \lambda(Y_r - V) + g\sin\theta_0 & 0 \\ Z_u & 0 & Z_w - \lambda & 0 & \lambda(V + Z_q) - g\sin\theta_0 & 0 & Z_n \\ 0 & L_v & 0 & \lambda L_p - A\lambda^2 & L_q & \lambda L_r + E\lambda^2 & L_n \\ M_u & 0 & M_w & M_p & \lambda M_q - B\lambda^2 & M_r & M_n \\ 0 & N_v & 0 & \lambda N_p + E\lambda^2 & N_q & \lambda N_r - C\lambda^2 & N_n \\ Q_{au} & 0 & Q_{aw} & Q_{ap} & Q_{aq} & Q_{ar} & 2\pi l\lambda + Q_{an} - Q_{en} \end{vmatrix} = 0$$

In forming the derivatives it must be remembered that $x_1$ and $z_1$ will be different for the two screws, and that the following must have their signs changed for the left-handed screw:—

$$X'_p,\ Y'_q,\ Z'_r,\ L'_u,\ M'_v,\ N'_w.$$

This determinant is capable of some reduction, but it does not seem profitable to attempt the reduction until we know whether any of the terms are negligible.

If the angle $A$ is zero $X_w$, $Z_u$, $Q_{aw}$ and $Q_{ar}$ vanish.

If, in addition, the c.g. of the whole machine lies on the axis of the screws the following derivatives disappear as well:—

$$X_q,\ Y_p,\ L_v,\ L_q,\ L_r,\ M_u,\ M_p,\ N_p,\ Q_{aq}.$$

When descending vertically with the screws free the torque is zero and $Z'_w = 0$, so that when $A$ and $z_1$ are zero, we also have $Z_w = 0$, $Z_q = 0$, $M_w = 0$, $M_q = M'_q$, $Q_{an} = 0$, $\theta = 90°$.

The equation has now reduced to:—

$$\begin{vmatrix} X_u - \lambda & 0 & 0 & 0 & 0 & X_n \\ 0 & Y_v - \lambda & 0 & 0 & \lambda(Y_r - V) + g & 0 \\ 0 & 0 & \lambda L_p - A\lambda^2 & 0 & E\lambda^2 & L_n \\ 0 & 0 & 0 & \lambda M_q - B\lambda^2 & M_r & M_n \\ 0 & N_v & E\lambda^2 & N_q & \lambda N_r - C\lambda^2 & N_n \\ Q_{au} & 0 & Q_{ap} & 0 & 0 & 2\pi I\lambda + Q_{an} \end{vmatrix} = 0$$

## II. Two Contrary-Turning Screws on Separate Shafts.

If we suppose the two shafts to be symmetrically placed with respect to the $xz$ plane, $y_1$ and $B$ will be equal and of opposite signs for the two screws. The derivatives can be written down from the general expressions given above, and we find that the extra axes which are introduced are $L_u$, $M_v$, $N_w$.

This completes for the moment the presentation of the stability equations. In a later paper I shall hope to give the results of some numerical investigations carried out with a view to throwing some light on the design of a practical helicopter.

## Errata in October issue:

p. 392, equation (5) for $\sin^2\phi$ read $\cos^2\phi$.
,, ,, (9) delete 2.
,, ,, (10) for $\pi^3$ read $\pi^3/2$.
p. 401, line 10, for $Z$ read $-Z$.
p. 402, for $\alpha$ read $\beta$.
,, ,, $Z$ ,, $-Z$.

# ASSOCIATE FELLOWSHIP EXAMINATION,

## September 25th-26th, 1922.

---

### EXAMINATION PAPER ON STRENGTH AND ELASTICITY OF MATERIALS AND THEORY OF STRUCTURES.

*Ten questions only to be attempted.*

1. Define stress, strain, elastic limit, yield point, permanent set, Poisson's ratio, and ultimate stress.

2. State Hooke's Law. Is this applicable to all materials? A duralumin tube 2in. external diameter and $\frac{1}{8}$in. in wall thickness encloses a steel tube $1\frac{3}{4}$in. external diameter and $\frac{1}{8}$in. wall thickness. The tubes are of the same length, and are rigidly attached to each other at the ends. If this composite tube is used as a tie rod what proportion of the tension is taken by each tube?

Young's modulus for steel $= 30 \times 10^6$lbs. per sq. in.
Young's modulus for duralumin $= 10.5 \times 10^6$lbs. per sq. in.

3. A steel bar 1 sq. in. in cross sectional area and 20in. long carries a load of one ton suspended from it. What is the maximum stress in the bar—

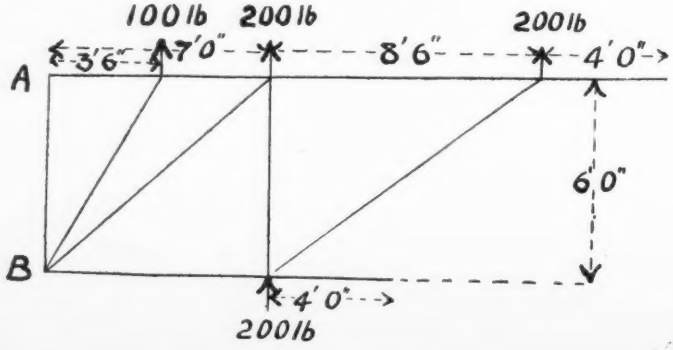
(*a*) When the load is applied gradually.
(*b*) When the load is applied suddenly.
(*c*) When the load is allowed to drop through $\frac{1}{8}$in. before it is taken on the bar.

4. Describe the tests you would apply to samples of silver spruce intended for aeroplane construction. Give the object of each test and the nature of the result you would hope to obtain.

5. What tests would you apply to a consignment of three-ply to ensure that it was suitable for aeroplane structural work?

6. A beam is pin-jointed at the ends and carries a uniformly distributed load of *w*lbs. per inch run. How would you determine the deflection at any point in the beam?

7. A pin-jointed beam is subjected to an irregularly distributed load. Given the loading diagram, show how the shear force and bending moment curves can be found.



8. The truss shown in the figure is supported at $A$ and $B$, and subjected to the loads indicated. Assuming the truss to be everywhere pin-jointed, find the loads in all members.

9. Discuss the method of failure of a long, thin strut. A streamline strut of silver spruce is 72in. long between pin joints. Its maximum axis is $4\frac{1}{2}$in., minimum $1\frac{1}{2}$in. in length, and it is parallel throughout its length. Calculate the axial load under which it will fail given that the moment of inertia of a streamline section is $BD^3/24$, where $B$ and $D$ are lengths of the maximum and minimum axes respectively, and that $E$ for spruce is $1.6 \times 10^6$lbs. per sq. in.

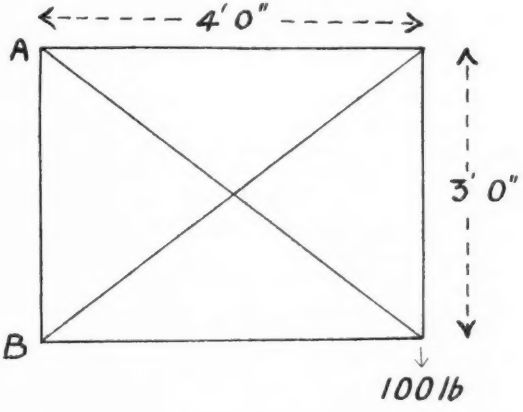
10. Show how you would calculate the maximum stress in a pin-jointed strut which carries an evenly distributed lateral load in addition to its axial thrust.

11. State the theorem of three moments in its usual form. Show how it may be used to calculate the reactions at the interplane struts in an aeroplane spar.

12. What modifications are required to the theorem of three moments to render it accurate for the calculation of spar stresses in an aeroplane? What approximate corrections are sometimes applied in practice in place of using the generalised equation?

13. Deduce an expression for the strain energy stored in a bar subjected to an axially applied tension or compression.

14. What do you understand by a redundant structure? Describe a method by which the stresses in such a structure can be calculated.




15. The pin-jointed structure shown is composed of steel members each $\frac{1}{4}$ of a sq. in. in cross sectional area. The frame is supported at $A$ and $B$, and a load of 100lbs. is hung on as shown. Neglecting any buckling of the strut members, determine the loads carried by each member of the frame.

## EXAMINATION PAPER ON AERODYNAMICS.

(Time, 3 hours.)

*Six questions only to be answered.*

1. Estimate the drag at an airspeed of 110 m.p.h. of 55ft. of streamline, interplane struts, 1.25in. thick, and 160ft. of streamline wires, 0.10in. thick, ignoring mutual effects. Make careful sketches (*a*) of a suitable section for the struts, to an enlarged scale; (*b*) of the attachment of strut and wires to a spar.

2. A disc, 10in. dia., held normal to the wind, gives a pressure distribution over its front and back face in accordance with the following table :—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Radius (in.) ... | 0 | 1.0 | 2.0 | 3.0 | 4.0 | 4.5 | 5.0 |
| Front (in. of water) | ø.40 | 0.39 | 0.37 | 0.34 | 0.28 | 0.18 | 0.00 |
| Back (in. of water) | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.14 | 0.00 |

Estimate the drag coefficient of the disc.

3. A seaplane of the parasol-monoplane type is 30ft. in span. Midway between the floats is a water vane, to assist getting off, geometrically similar to the wing. The leading edge and chord of the vane are parallel to those of the wing, and it is 2ft. 6in. in span. Neglecting any mutual effects, explain why the lift of the vane does not remain a constant fraction of the lift of the wing during flight. Also show that while the machine taxies with the vane sufficiently deeply immersed in water having 1/12th the kinematic viscosity of air, the reverse is true, whatever the speed. In the latter case, calculate the constant relation between the lifts, taking water as weighing 64lbs. per cu. ft. Establish any formula to which you may refer.

4. (*a*) A model to twice full scale of a wing tip float, tested in a wind channel, gave the following results :—

| | | | | |
|---|---|---|---|---|
| Speed (ft. per sec.)... | 60 | 70 | 80 | 90 |
| Drag (lbs.) ... ... | 2.07 | 2.65 | 3.30 | 4.05 |

Find the air-drag of the float at 150ft. per sec.

(*b*) A flying boat hull is thought to have a water-drag of 4,000lbs. at 20 m.p.h. At what speed should a 1/20th scale model be towed in a tank to check this? What drag would the model then have if the estimate for full size were correct?

5. Carefully sketch the sections of two wings; one suitable for high speed, the other giving a high maximum lift. Show in a single diagram marked with proper scales, the curves of $k_L - a$ and $L/D - a$ you would expect from a model test in each case. Making use of your diagram, explain any case of a prospective machine in which the high lift wing might be preferred to the other.

6. An aeroplane, weighing 2,600lbs., has a speed range of 51 to 120 m.p.h. at low altitude. The thrust horse-power available falls from 180 at ground to 85 at 20,000ft. The wings have the following characteristics :—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Lift coef. | .090 | .165 | .252 | .310 | .422 | .480 | .500 (max.) |
| Lift/drag | 9.3 | 13.0 | 13.0 | 12.3 | 9.8 | 9.0 | 6.0 |

Estimate the maximum rate of climb at ground level and at 20,000ft. (Relative density = 0.53.) Ignore airscrew effects.

7. Discuss the economic advantage of flying at considerable altitude. An aeroplane is flown due north at an altitude of 10,000ft. ($\rho = .00175$), at an indicated airspeed of 90 m.p.h. There is a steady north-westerly wind of 30 m.p.h. Find the distance covered in one hour.

8. Briefly discuss the question as to whether an aeroplane, initially flying in a straight horizontal path, gains or loses speed on a properly banked turn. An aeroplane is put into a horizontal circular path of 200ft. radius. There is no side-slipping and the steady speed becomes 80 m.p.h. What record would the accelerometer make of the manœuvre?

9. A two-bladed airscrew turning at 1,260 r.p.m. is required to drive an aeroplane at 130ft. per sec. At 3ft. 9in. radius the angle of incidence of the section of the blade is to be four degrees, and the local thrust 110lbs. per ft. run. At what angle must the section be inclined to the plane of rotation? (Assume

that the ratio of the inflow factor to the outflow factor is 0.35, and neglect rotation of the airstream.)

10. A glider is found to have a tendency to get into a spin. An attempt to correct this produces spiral instability. Describe each form of instability, suggesting what alteration was probably made. State what further steps you would take to ensure satisfactory flight.

## EXAMINATION PAPER ON HEAT ENGINES.

*Answers are not expected to all the questions, but full answers to six should be attempted, and should include the first three questions on the paper.*

1. Why does the power of an internal combustion engine fall off with increase in altitude? What methods have been proposed to counteract this decrease? Discuss briefly the respective merits of the various proposals.

2. Describe briefly one form of device fitted to aero engine carburettors to enable the pilot to control the mixture strength at different altitudes. Why is such a device fitted?

3. Why is a high mechanical efficiency very desirable in any engine and particularly in an aero engine? An internal combustion engine, when developing 400 brake horse-power, was found to have a mechanical efficiency of 88 per cent. It was also found that the piston friction alone amounted to 50 per cent. of the total losses. Estimate the h.p. wasted in piston friction, and express the result in B.T.U.'s. What becomes of this heat?

4. Give the equation correcting pressures and volumes for adiabatic compression and expansion of a gas. The following data relates to the compression of 100 volumes of a gas :—

| Volumes. | Pressures lbs. sq. in. (gauge). |
|---|---|
| 100 | 0 |
| 50 | 25·3 |
| 30 | 65·3 |

Show how to determine the value of the index "$n$" from the above data.

5. It is found that the pressures obtained by ignition of a petrol-air mixture are considerably lower than those calculated from the simple gas laws. How do you account for this?

6. Define indicated thermal efficiency, brake thermal efficiency and relative indicated thermal efficiency. An engine running on a brake test at 1,500 r.p.m. lifts a net weight of 117lbs. at a brake arm radius of 3ft., and consumes one pint of fuel in 70 seconds. The lower heat value of the fuel is 18,900 B.T.U. per pint, the compression ratio 5 : 1 and the mechanical efficiency 87 per cent. Find the indicated thermal efficiency, brake thermal efficiency and the relative indicated thermal efficiency.

7. Discuss the balance of an eight-cylinder 90 deg. Vee type engine with all the cranks in one plane. In such an engine the reciprocating weight for each cylinder is 10lbs., the stroke being 6in. and the connecting rod length 12in. What is the magnitude direction and frequency of the secondary out of balance force at a speed of 1,800 r.p.m.? Assume that the connecting rods are all attached directly to the crank pins.

8. Show by means of a clock-face diagram the usual disposition of the cranks of a six-cylinder four-stroke cycle petrol engine, and give a suitable firing order for the six cylinders.

9. Why is it particularly desirable that, in six-cylinder line and twelve-cylinder Vee type engines, the engine structure shall have considerable longitudinal stiffness considered as a beam? Compare various types of engine structures from this point of view, including British and German practice.

10. What are the sources of loading on the big end bearing of a single acting four-stroke cycle stationary engine? How is the loading affected by variations in speed and load? Assuming constant engine speed and considering the conditions which exist when the engine is running on full throttle at ground level and at a considerable altitude, do any of the sources of loading vary in amount, and if so, which and in what sense?

11. What are the considerations which lead one to use (*a*) multiple inlet valves and (*b*) multiple exhaust valves?

12. In high duty engines such as aero engines having plain bearings, it is common practice to circulate considerable quantities of lubricant through the bearings. What are the reasons for this procedure? What is meant by a "dry sump" lubrication system, and what are the reasons underlying its adoption on aero engines?

13. What is understood by the expression "bad distribution" in connection with the running of a multi-cylinder petrol engine? What influence has bad distribution on the performance of an engine?

14. Discuss the relative advantages and disadvantages in the use of aluminium alloys, cast-iron, and steel for (*a*) the pistons, and (*b*) the cylinders of water and air-cooled aero engines.

15. Discuss, from the aircraft designer's point of view, the various types of engines (straight line, Vee, radial and rotary) as applied to (*a*) commercial and (*b*) military service, indicating the types of machines for which each type of engine is more particularly suited.

## ASSOCIATE FELLOWSHIP EXAMINATION, PART II.

### MATHEMATICS.

1. A cone of height $h$ and circular base of radius $a$ is composed of material of density $\rho$. Find the depth to which it will float vertex downwards in a fluid of density $\sigma$. Hence show that if it be depressed vertically by a small amount beyond this depth it will oscillate when released, and find the period of a small oscillation.

2. State Newton's Second Law of Motion. A body is projected with an initial velocity $V$ in a medium in which the only force that operates is a resistance of magnitude $kv^n$ along the line of motion, $k$ and $n$ being constants and $v$ the velocity of the body. Show that the body will not come to rest in a finite distance unless $n$ is less than 2.

3. Show that in the motion of a particle of mass $m$ in a plane, under given forces, the latter must be equivalent to $mdv/dt$ along, and $mv^2/\rho$ at right-angles to the path, $\rho$ being the radius of curvature and $v$ the velocity at the point. A closed elastic band fits tightly round the circumference of a wheel of radius $a$, the tension being $T_0$. Find the angular speed of the wheel in order that the band may just leave it.

4. Explain the terms *dimensions* and *homogeneity of dimensions* with reference to the terms of a physical equation. A body is projected with velocity $V$ under an acceleration $A$ whose magnitude and direction are variable and

unknown. Show that if the speed of the body is $v$ at any instant when a distance $s$ has been traversed then the acceleration $A$ must be expressible in the form—

$$A = V^2/s \, f(v/V),$$

where $f$ is some unknown function. Verify this formula for the case of a body projected vertically under gravity.

5. Sketch roughly the graph of the expression—

$$y = x\,(x^2 - 1)/(x^2 - 4);$$

hence determine the number and signs of the real roots of the equation—

$$x\,(x^2 - 4) - (x^2 - 1) = 0.$$

6. Define *direction cosines*. Prove that if two lines have direction cosines $(l_1, m_1, n_1)$ and $(l_2, m_2, n_2)$ the angle $\theta$ between them is given by—

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2.$$

If possible without using tables, find in degrees the angle between the two lines having direction cosines proportional to (1, 2, 2) and (2, 4.1, 3.9).

7. State and prove the formula for integration by parts. Evaluate—

(1) $$\int_0^{\pi/2} x \sin x \, dx.$$

(2) $$\int_0^{\infty} dx\,(x^2 + a^2)/(x^2 + b^2)\,(x^2 + c^2).$$

8. State and prove the conditions that must be satisfied by the maximum and by the minimum values of the function $y = f(x)$. Investigate the turning points of the functions—

(i) $(x^2 + ax + b)/(x^2 + 1)$. (ii) $(\sin x)/x$.

9. Write down Maclaurin's Theorem. Expand cosh $x$ in ascending powers of $x$ and hence show that—

$$\cos ix = \cosh x \text{ where } i^2 = -1.$$

10. The curve $y = f(x)$ is rotated round the axis of $x$. Find an expression for the volume enclosed between the resulting surface and the planes $x = x_1$ and $x = x_2$. Deduce the volume of a right circular cone of height $h$ and radius of base $r$.

11. Explain precisely what is implied by the phrase "neglecting terms of higher order than the first." If a differential equation is so simplified by applying this process that it becomes capable of simple solution, how far would you be justified in relying on the results obtained by such a solution?

12. State carefully any problem represented by a differential equation of the type—

$$d^2y/dt^2 + 2k\,dy/dt + n^2y = 0$$

where $k$ and $n$ are constants. Solve the equation completely in the case cited and interpret the results.

## NAVIGATION AND METEOROLOGY.

*Not more than seven questions to be answered.*

1. Describe meteorological conditions in which fog might be expected at aerodromes situated (*a*) on a high level plateau, (*b*) on flat, low-lying ground.

2. The centre of a rather deep cyclone is situated over the North of Scotland, and is moving eastwards. Describe weather conditions over Western Europe during the following 24 hours.

3. Write a short essay upon *either* (*a*) variation of wind with height, *or* (*b*) variation of temperature with height.

4. Write short notes on the following :—

(*a*) " Gradient " or " geostrophic " wind.
(*b*) Line squalls.
(*c*) The stratosphere.
(*d*) Haze and visibility.
(*e*) Meteorological anemometers.
(*f*) The errors of airspeed indicators, when used at heights.

5. Meteorological information gives the wind at 5,000ft. at 20 m.p.h., from the south-west. An aeroplane has to fly to an aerodrome 100 miles due east from the starting point. Show how the time occupied by the flight and the required compass bearing could be calculated.

6. Describe any type of turn indicator, and discuss the advantages of these instruments for use in flying through cloud.

7. Describe briefly the working of any type of sextant for use on aeroplanes, and show, in detail, how to calculate your position from sextant observations of the sun or stars.

8. Write short notes on the following :—

(*a*) Great circle route.
(*b*) Somner lines.
(*c*) Magnetic variation.
(*d*) Magnetic deviation.
(*e*) Errors of altimeters.
(*f*) Course setting instruments.

9. Describe fully any method of determining your position when flying in an aeroplane by means of direction-finding wireless.

10. Discuss the causes which give rise to the unsatisfactory working of compasses when used in an aeroplane, and why are similar troubles not noticed on ships?

## METALLURGY.

*Questions one and two, and four of the remainder are to be answered.*

1. Explain the meaning of the terms normalising, hardening, tempering, blueing, as applied to steel. Give the normalising and hardening temperatures of steels of the following compositions :—

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Carbon, per cent ... | 0.10 | 0.40 | 0.30 | 0.30 | 0.35 | 0.25 |
| Manganese, per cent.... | 0.50 | 0.60 | 0.35 | 0.65 | 0.45 | 0.50 |
| Nickel, per cent. ... | nil | nil | 3.75 | 3.25 | 0.50 | 4.30 |
| Chromium, per cent. ... | nil | nil | 0.85 | 0.10 | 12.5 | 1.20 |

State the quenching medium you would employ in treating (*a*) a 3in. round bar, (*b*) a ½in. round bar of each of the above steels.

2. Give the approximate tensile and impact properties of 1in. round bars of the six steels given in Question 1 after cooling in (*a*) air, (*b*) water, (*c*) oil. Give the approximate maximum stress, elongation per cent. and impact value of steels B, C, and D after hardening in oil in 1in. round bars from the temperature you think best, followed by tempering at 200deg. C., 400deg. C., 600deg. C. respectively. Are the impact values of the six steels given in Question 1 affected by the mode of cooling from these tempering temperatures?

3. State the essential features of the operation of case-hardening. Give the approximate chemical composition of three case-hardening steels, and state any variation from the procedure you have described when producing a case-hardened article in the three steels you select.

4. What is duralumin? State its approximate chemical composition, its mechanical properties when properly treated, and the heat treatment that should be given to it in order to produce these properties.

5. What are the principal alloys of aluminium, either wrought or cast, used in aircraft? Give the approximate mechanical properties of each alloy and its specific gravity.

6. What are the predominant features of (*a*) cast, (*b*) forged or drop forged, (*c*) rolled metals? What is the effect of micro-structure upon the mechanical properties of steel?

7. What materials would you employ for five of the following purposes:—

(*a*) An exhaust valve in a hot engine.
(*b*) Dip-brazing of steel fittings.
(*c*) Fork ends.
(*d*) Tie rods.
(*e*) Gudgeon pins.
(*f*) Reduction pinions.
(*g*) Radiator tubes.
(*h*) Petrol piping.
(*i*) Petrol tank.

Give the approximate chemical composition of the material chosen for each part.

8. Give the approximate chemical composition and general mechanical properties of three of the following:—

(*a*) Muntz metal.
(*b*) Delta metal.
(*c*) Naval brass.
(*d*) Cast phosphor bronze.
(*e*) Gunmetal.

# REVIEWS.

**Grundlagen der Flugtechnik.** Dr. H. G. Bader.

The aim of this book is to give an account of all the aerodynamic calculations required for the design of an aeroplane, other than those dealing with the strength of the structure, and on the whole this object is carried out successfully. The treatment of the problems is almost purely theoretical, and experimental results are introduced only to indicate the accuracy of the theory or to provide numerical values for the constants which occur in the formulæ.

Starting with a rather inadequate account of the theory of the lift and drag of wing structures, the author passes on to a full treatment of the performance of an aeroplane, and of the choice of design to secure high speed, high rate of climb or great endurance. The balance of the aeroplane is next considered in detail, together with the problem of securing weathercock stability. The author advocates that the stability should be positive in this sense, but that the margin of stability should be small. This is followed by a full treatment of the longitudinal and lateral stability of the aeroplane, which, however, contains no new features. Finally there is a most interesting chapter on the problems of getting off and landing.

On the whole the treatment of the various problems is most satisfactory, and although often differing in form from the usual English practice, the work is really identical in all essential points. The only real gap in the book is the failure to give any account of the theory of the airscrew, this work being replaced by a couple of diagrams of airscrew characteristics. This gap is the more noticeable as a very full account is given of the effect of the airscrew slipstream and of its effect on the angle of downwash, but it is the only gap in a very full treatment of the aerodynamic problems of aeroplane design.

**Direction and Position Finding by Wireless.** R. Keen. Wireless Press.

A book dealing exclusively with directional wireless is welcome, particularly as most of the recent published information on this subject has appeared in the proceedings of scientific societies and is therefore not readily available, nor likely to be fully understood by the general reader. The book is very well illustrated by diagrams and photographs, and the subject is presented in such a manner that it should be quite clear to those whose knowledge of wireless is limited.

The Author has adopted an excellent system for reference in the text to published articles dealing more fully with a particular point. At the end of the book a bibliography is given, arranged in chronological order, and the items are consecutively numbered, these numbers being quoted in the text when reference to the paper would give additional information. This system could be adopted in other technical works with advantage.

The book deals chiefly with the Marconi-Bellini-Tosi system, particularly as applied to shore and ship installations, and though these descriptions are very interesting, the chapter on aircraft installations will be of the greatest value to readers of the AERONAUTICAL JOURNAL. This chapter is unfortunately one of the shortest in the book, and the aircraft sets are therefore not described in great detail. The use of the wing coil system for taking bearings or homing on a sending station is discussed and the effect of drift explained. The Robinson

system, used by the Royal Air Force, is briefly described and illustrated by photographs, and particulars are given of a Marconi-Bellini-Tosi set for aircraft.

Very little space is allotted to the directional transmitter or "Radiophare," and this is to be regretted as this system may well prove to be of the greatest value for aircraft navigation.

The chapters dealing with maps and position finding explain the use of directional wireless for navigation in a simple and interesting manner. The way in which polarisation, reflection and refraction of the electro-magnetic waves give rise to errors, and the methods employed to reduce these errors, are dealt with in a chapter headed "Night Effect," and by the aid of numerous illustrations this rather complicated subject is rendered easy to understand.

The final chapter gives notes on field and nautical astronomy, chiefly for the guidance of those erecting ground stations, and it is in this chapter that the only error noticed in the book occurs. The point X in Fig. 244 should be on the ecliptic as it is the true position of the sun on the celestial concave.

The Author has completed the task he set himself in a very satisfactory manner, and if the criticism is made that too little space has been devoted to aircraft installations, the Author would be perfectly justified in replying that he has allotted space in proportion to the importance of the subject. There is no doubt that directional wireless is not used on aircraft to any great extent at the present time, but it is to be hoped that the promise shown by this adjunct will one day be realised and in some future edition of this book the section dealing with aircraft sets may rival in importance the chapters dealing with the installations in surface craft. Meanwhile, the book can be confidently recommended to those interested in the subject.

# CORRESPONDENCE.

Cambridge Road,
South Farnborough.

*To the Editor of the* AERONAUTICAL JOURNAL.

DEAR SIR,—The article on helicopters by Mr. J. Case, in the October issue, mentions a serious discrepancy in calculating the thrust at zero advance by the multiplane interference theory, according as the corrections are estimated from the experimental data of R. and M. 639, or are calculated by the vortex theory of aerofoils as in R. and M. 752. In the first case a thrust of 2,000lb. per blade was obtained, while in the second case the value was 370lb., and this last figure was in agreement with model tests of the airscrew. The following notes are an attempt to account for this discrepancy.

The experimental data of R. and M. 639 were obtained with aerofoils of aspect ratio 6, while the calculations for the helicopter were based on the actual aspect ratio 3 of the blades. Calculations with aspect ratio 6, however, would give a thrust of 700lb. instead of 370lb. as obtained with aspect ratio 3.

In the second place the calculations made on the vortex theory have shown that, owing to errors of extrapolation, the experimental results of R. and M. 639 may be considerably in error. This error depends on the span-gap ratio of the series of aerofoils, being negligible when this ratio is unity and rising to 5 per cent. when the ratio is 6. In the case of the helicopter under discussion this ratio was as high as 33 and the error of extrapolation can therefore be estimated as of the order of 25 per cent. If this correction is made the thrust per blade is found to be 750lb. instead of 2,000lb. and this figure is in reasonable agreement with the estimate made by use of the vortex theory.

It appears from this discussion that if the multiplane interference theory is to be used with success the correction factors should be calculated by the vortex theory as in R. and M. 752 and that the mean aspect ratio of the blades should be used.

Yours faithfully,
H. GLAUERT.